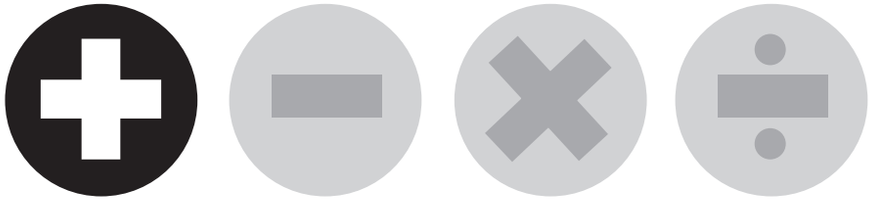
THINKING STRATEGIES: ADDITION

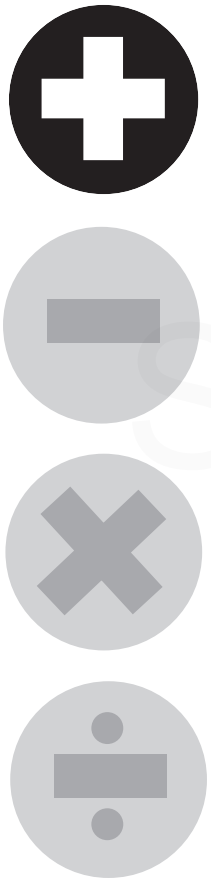
BUILDING MASTERY OF ADDITION FACTS



CELIA BARON

MathImagine

CONTENTS



Introduction 1

Level 1: Facts with 0, 1, 2, and 3 5

Level 2: Facts with 9 and 10 29

Level 3: Double Facts 49

Level 4: The Near-Doubles 71

Level 5: Facts with 8 91

Level 6: Remaining Facts 109

Appendix A: Teacher Resources 131

- Addition Grid 132
- Power Facts/ Summaries of Thinking Strategies 133
- Progress Report for Students 139
- Self-Assessment Progress Report for Students 140
- Letter to Parents/Guardians 146

Appendix B: Partner Bingo 147

- Partner Bingo 148

Appendix C: Challenge Facts 179

- Challenge Facts 180

Appendix D: Playing Cards 201

- Playing Cards 202

Appendix E: Templates 217

- Ten-Frame Mat 218
- Working Ten-Frames 219
- Mini Ten-Frames 222
- Ten-Frame Train 223
- Number Cards 224
- Dot Cards 230

Appendix F: Answer Keys 233

- Student Activity Sheets 234
- Partner Bingo 247

INTRODUCTION

PROGRAM GOAL

Thinking Strategies: Addition is a program designed to help students master the basic addition facts. The program accomplishes this goal by (1) introducing the facts in logical rather than numerical order and (2) using the commutative or turnaround property (teaching facts like 2 + 3 and 3 + 2 together). The program uses the mathematical processes of communication, connections, reasoning, representation, and problem solving to encourage learning. Teachers can use *Thinking Strategies: Addition* with an entire class, small groups, or individual students.

WHAT ARE THE BASIC ADDITION FACTS?

The basic addition facts are combinations, like 5 + 8 or 9 + 3, in which the addends (numbers being added) are less than 10. Because our number system is a base ten number system however, the program also includes addition facts in which the numbers being added are 10, as well as the numbers that are less than 10.

WHAT IS MASTERY OF THE BASIC FACTS?

In his book, *Elementary and Middle School Mathematics*, John Van de Walle defines mastery of a basic fact as a quick response time of less than three seconds. When a student is able to correctly respond to a fact automatically and without thinking, he/she has mastered that fact.

Building mastery of the basic facts involves the following four stages:

Stage 1: incorrect response/an inappropriate thinking strategy

Stage 2: correct response in more than three seconds using an appropriate thinking strategy

Stage 3: correct response within three seconds using an appropriate thinking strategy

Stage 4: correct response that is automatic and occurs without thinking

Mastery develops with practice (reviewing a variety of facts or procedures). The practice is provided in the program through student activity sheets, Power Facts, Partner Bingos, Card Games, and Challenge Facts.

Mastering the Basic Facts: The Latest Research

Van de Walle states that all students are able to master the basic facts if they follow three steps:

1. develop a strong understanding of the operations and number relationships
2. develop efficient thinking strategies for fact retrieval
3. practice the use and selection of those strategies

WHAT IS A "THINKING STRATEGY"?

A thinking strategy is a way of thinking that helps complete a fact *quickly*. For a strategy to be a thinking strategy, it must be done *mentally*, and it must be efficient.

The more senses you can involve when introducing the facts, the greater the likelihood students will remember how to complete the facts. Different strategies work for different students. By providing a variety of strategies, students can choose what works best for them. Some strategies are visual – for example, the special pictures that are used to complete the double facts. Some strategies are auditory and involve rhymes. Many of the strategies involve patterns and connecting facts that students have yet to learn with facts they already know how to complete.

PROGRAM LEVELS

Thinking Strategies: Addition is divided into six levels. Each level begins by having the students model the facts introduced in that level. These models are visual representations of the facts and help students understand why a fact is completed the way it is. Once the students have developed and understand the strategies of that level of the program, they practice the strategies. The facts are then incorporated with the facts introduced in previous levels. At the end of each level, a Level Challenge activity helps students identify and apply thinking strategies to the facts introduced to that point of the program.

Level 1: The first level examines the basic addition facts with an addend of 0, 1, 2, or 3. It introduces the concept of addition, the commutative or turnaround property, and many of the models used in the program. Four important relationships students should develop for the numbers 1 through 10 are discussed, and games and activities to practice these relationships are provided. In this level, the thinking strategies for the facts with an addend of 0, 1, 2, or 3 are introduced.

Level 2: In the second level, the basic addition facts with an addend of 9 are examined. Three important relationships students should develop for the numbers 10 through 20 are discussed, and games and activities to practice these relationships are provided. The thinking strategies for facts with an addend of 9 involve anchoring to the number 10.

Level 3: The third level examines the doubles, which are basic addition facts that have the same addends. A number of thinking strategies are introduced for completing the doubles. One strategy involves special pictures for the doubles.

Level 4: The fourth level examines the near-doubles, which are basic addition facts with addends that differ by one. Many of the near-double facts were introduced in the first three levels of the program. The near-double facts can

be related to the double facts; the near-double thinking strategy based on this relationship is developed in this level.

Level 5: In the fifth level, the basic addition facts with an addend of 8 are examined. Only three facts with an addend of 8 and their turnarounds have not been introduced in the program by level 5. These facts are among the most difficult to learn. A variety of thinking strategies are developed for these facts.

Level 6: The sixth level examines the remaining basic addition facts. There are only three remaining facts and their turnarounds left to introduce. A variety of thinking strategies are developed for these facts.

PROGRAM COMPONENTS

Teacher Lessons

The lessons give clear directions for working through the program and provide instructions that teachers can use to help students master the addition facts. The lessons involve working with models and developing number relationships. Students are encouraged to find as many strategies as possible for completing the facts. With many strategies to choose from, students soon find the strategies that work best for them.

Class Discussion

Class discussions are an integral part of the program. The following are some statements and questions that can be used to encourage student participation in class discussions:

- Explain and justify your answer.
- Explain your answer in another way.
- Can someone else explain the answer in another way?
- As a group, describe a different way to find the answer.
- If you did not know how to complete this fact, what thinking strategy would you use?
- What is another fact that you can complete this way?
- How can you model this fact with a picture?
- How can you model this fact with another picture?
- How does this model explain the fact?
- Make up a story problem for this fact.
- How does your story problem illustrate the fact?

Practice for Students

The student activity sheets support understanding of number relationships and give students practice with the basic addition facts. The activity sheets, which consist mainly of secret messages, line designs, and puzzles, are intended to be fun and engaging. They have the added advantage of being mainly self-correcting.

The program makes it easy to monitor the progress of students. A level challenge activity is at the end of every level. The Level Number Challenge checks each student's ability to identify and apply the appropriate thinking strategies to the facts introduced to that point in the program. Teachers can keep track of students' progress in other ways. As the students complete each level of the program, have them make a list of the facts they have yet to master. Have the student practice these facts both at school with a classmate and at home with a parent. When the student is able to complete these facts, check them off on the student's progress report (page 139). While students are working on the activity sheet that accompanies each lesson, teachers can also check which facts students can complete.

At the completion of a level, teachers will find suggestions for more practice. Students who have not yet mastered the facts introduced in that level can still continue to the next level of the program. They will have more opportunities to practice these facts in the levels that follow.

Addition Grid

At the beginning of the program, hand out a copy of the addition grid to each student (page 132). In each level of the program, students are asked to fill in the grid for the facts they have been introduced to. Teachers may also want to keep a master grid.

Power Facts

There are six sets of Power Facts (pages 133-138) – one set for each level in the program. Each set has 20 facts. The facts consist of those that are introduced in that level plus some of the more difficult facts from previous levels. The Power Facts are asked only after the thinking strategies for those facts have been fully developed.

The Power Facts are intended to support the learning of the addition facts. Ask these facts in class each day. Allow students no more than three seconds to complete each fact on their answer sheet, and then read out the addition fact and its sum (answer). If the students have been unable to complete the fact in three seconds, or if they have completed it incorrectly, have them write the completed fact on their answer sheet. Wait 5-7 seconds between facts to give the students time to process the fact. Students can measure their improvement each day as they are able to complete more facts correctly.

Each set of Power Facts is given in rows and columns. It is important to change the order in which the facts are asked. One day ask them vertically, the next day horizontally, then from top to bottom, and another day from bottom to top. Also, change the order of the addends in the facts.

Partner Bingo

Partner Bingo (page 147) is a two-player game. Students can play it in class or at home with parents and siblings. Players have a bingo card and 16 facts that they take turns completing. As they complete each fact in order, they shade in one square on their bingo card. The same number might appear in more than one box on the bingo card, and the students must choose which to fill in. The first player to complete a row, column, or diagonal is the winner.

Partner Bingo provides practice with the basic addition facts in a fun game setting. The games are a positive way for parents to determine which addition facts their children are able to complete and which facts need more practice. There are Partner Bingo games in each level of the program.

Challenge Facts

The Challenge Facts (page 179) consist of addition facts for each level of the program. Recent research does not support using timed tests to help students master the facts. Some students become very anxious when faced with timed tests, and this can affect their sense of their ability to do mathematics.

The Challenge Facts are included in the program as a diagnostic tool. Teachers can use these sheets to find out which facts the students have mastered and which facts they have not mastered. The students can circle the facts on the Challenge Facts sheets that require more practice. There are no time limits for completing the Challenge Facts. Discourage students from guessing. Completing a fact incorrectly reinforces the incorrect answer.

There are two types of Challenge Facts, A and B, in all levels except Level 1. Type A Challenge Facts consist of the facts introduced in a particular level. Type B Challenge Facts contain all of the facts introduced in the program up to that level. Students can use the Level A Challenge Facts to practice the facts developed in that level. Students can use the Level B Challenge Facts to practice the facts introduced to that point. Students can also use the Level B Challenge Facts to practice identifying facts. To do this, students can draw shapes or use coloured pencils to identify facts that are completed with the same strategies.

Playing Cards

Playing Cards (page 201) are provided for each level of the program, with an extra deck for the first level. A "deck" of playing cards consists of forty cards, ten sets of four cards each. Each set consists of four different ways to represent a fact. The cards can be laminated. The students can play various card games with these decks, including War, Fish, Snap, and Concentration. Two extra "decks" are included: one containing dot patterns; the second with representations of the numbers 1 through 10.

Ten-Frames

The ten-frame is a powerful model for completing the addition facts. Various sizes of ten-frames are included in the program. Encourage students to use these ten-frames to model and complete addition facts. As well, a ten-frame train, together with passengers, is provided in the program (page 223).

Number Cards

Number Cards for the numbers 1 through 20 are included in the program (pages 224-229).

Teacher Assessment

The intent of the program is not only to have the students master the basic facts, but also to have them do so in a positive manner. The evaluation and assessment should reflect this. Continuous assessment and evaluation allow recognition of student achievement. When students are successful in mastering the facts, they feel good about themselves and about their ability to learn. A powerful assessment tool is to have the students keep track of the facts that they have mastered. Doing so allows the students to see how much they are learning and to feel a sense of accomplishment.

The National Council of Teachers of Mathematics recommends the integration of assessment and instruction. The program *Thinking Strategies: Addition* supports this integration. The student activity sheet that accompanies each lesson allows teachers to assess whether or not the students are following the program. As well, many of the activities are self-correcting and allow the students to know if they have understood the lessons.

Teachers can also use the Power Facts to assess the progress of students. Students can hand in their Power Facts answer sheets each day, or they can track their own progress. With either method, the teacher can use the results to complete the students' progress reports. A student progress report for each level is included in the program (page 139).

Student Self-Assessment

In this program, students can track which facts they have mastered and which facts they have yet to master. As they complete each level of the program, have the students write about the facts they are learning in their journals. Ask them to indicate whether or not the facts in the level that they are working on are easy for them to complete. Have them explain why or why not. They may want to list both the facts they have mastered and the ones they have yet to master. Have them describe thinking strategies for the facts they have yet to master. If students need help in identifying thinking strategies, work with them to find the strategies. A student self-assessment progress report for each level is included in the program (pages 140-145).

Using Individual Response Boards

Student response boards can be either a chalkboard or a dry-erase board made of laminated sheets of cardboard. In classrooms where each student has a response board, the students can write the products of the facts on their boards. Teachers can check off the students' progress reports as the students show the answers on their boards.

Teachers in classes with no response boards can have students mouth the answer. This method of response provides students with opportunities to answer a question, and it allows the teacher to monitor the progress of his/her students.

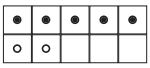
Parental Guidance

Teachers might want to involve parents in the learning process. Power Facts and thinking strategies can be sent home with students so that they can practice the facts with their parents as they are being taught the facts in class. Parents can also help their children by playing Partner Bingo with them.

MODELS USED IN THE PROGRAM

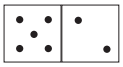
The following models are used in the program:

- Ten-Frames: A ten-frame is an array of 2 rows and 5 columns in which counters or dots represent numbers. The top row is filled in first, beginning on the left. Once the top row is full, the second row is filled in, again beginning on the left. For example, the fact 5 + 2 is illustrated by the following ten-frame:

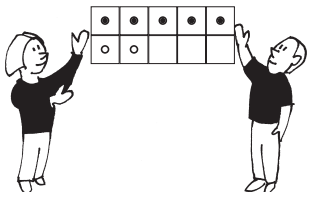


- Cube Trains: A cube train consists of a row of cubes. The cube train modelling the fact 5 + 2 consists of 5 cubes of one colour and 2 cubes of another colour.

- Dominoes: The domino modelling the fact 5 + 2 consist of 5 dots on one side of the domino and 2 dots on the second side.



One or two addition buddies appear on each student activity sheet. The buddies illustrate a fact introduced in that lesson of the program. The fact is usually the first fact of the activity sheet.



LANGUAGES IN ADDITION

Van de Walle explains that it is useful to think of number sentences, models, and word problems as three separate languages. For example, the fact 5 + 2 can be expressed in the following ways:

- A number sentence would be 5 + 2 = 7.
- A model could be a ten-frame that contains 5 dots in the first row and 2 dots in the second row.
- A word problem could be a story problem involving 5 pencils and 2 pencils.

When students complete a fact, having them translate from one language to another helps them develop operation meaning. Understanding the concept of addition is the foundation for mastering and applying the addition facts. Many of the facts of each level are introduced with story problems. As well, students are encouraged to create their own story problems throughout the program.

DIFFFERENT CLASSES OF ADDITIVE STRUCTURES

Van de Walle explains that most researchers identify four categories of addition problems. Two examples of story problems from each category are given. The second set of problems involves humour. These categories, based on the kind of relationships involved, are the following: join problem, result unknown; separate problem, initial unknown; part-part-whole problem, whole unknown; compare problem, larger unknown.

- Join Problem: Result Unknown

  Leah has 5 pencils. Her friend Cathy gives her 2 more. How many pencils does Leah have altogether?

- Separate Problem: Initial Unknown

  Brian had some marbles. He gives 4 to Garrett. Now Brian has 7 marbles. How many marbles did Brian have before he gave some to Garrett?

- Part-Part-Whole Problem: Whole Unknown

  Aisha has 2 dogs and 3 turtles. How many pets does she have altogether?

- Compare Problem: Larger Unknown

  Danica has 3 more books than her sister Candace. If Candace has 8 books, how many books does Danica have?

The following story problems use humour to illustrate the different categories of additive structures.

- Join Problem: Result Unknown

  A giraffe has 5 bowties. A friend gives the giraffe 2 more bowties. How many bowties does the giraffe have altogether?

- Separate Problem: Initial Unknown

  An elephant has some watches and gives 3 to an antelope. Now the elephant has 5 watches. How many watches did the elephant have before he gave some to the antelope?

- Part-Part-Whole Problem: Whole Unknown

  A rabbit is holding 8 carrots in one paw and 5 carrots in the other paw. How many carrots is the rabbit holding altogether?

- Compare Problem: Larger Unknown

  Baby Bear is so hungry she eats 2 more bowls of porridge than Papa Bear does. If Papa Bear eats 4 bowls of porridge, how many bowls of porridge does Baby Bear eat?

LEVEL 1: FACTS WITH 0, 1, 2, AND 3

LEVEL 1 OVERVIEW

In Level 1, students are introduced to all of the basic addition facts with an addend of 0, 1, 2, or 3, and to number relationships they can and should develop with numbers 1 through 10. These number relationships are the basis of building an understanding of the operation of addition and of the thinking strategies introduced in this program. The importance of these relationships cannot be overstated.

Number Relationships 1 Through 10

In his textbook, *Elementary and Middle School Mathematics,* John Van de Walle lists the following four relationships that students can and should develop with numbers 1 through 10:

- spatial relationships
- one and two more, one and two less
- anchors, or benchmarks, of 5 and 10
- part-part-whole relationships

These relationships are very important in building an understanding of the addition facts; check that your students have developed them. If your students need more practic with these relationships, consider the activities suggested in the first level of the program.

Spatial Relationships

Spatial relationships refer to the ability to recognize sets of 1 to 10 objects in patterned arrangements and to tell how many there are without counting. To determine if your students have developed this relationship, play Dot-Pattern Flash, which uses the pattern arrangement of 1 to 10 dots (page 230): Present students with a patterned arrangement of dots for a few seconds. Have them write the number of dots they see on their response boards. This exercise allows you to check whether students can tell how many dots there are without counting. Play until students do not have to count the number of dots.

Refer to the first set of "playing cards" (pages 202-203). The students can use these cards to play games of Snap, War, Fish, and Concentration. The game of dominoes also can provide the students with practice of patterned arrangements of dots.

The One- and Two-More, One- and Two-Less Relationships

Other relationships that students should develop for the numbers 1 through 10 are one and two more, one and two less (John Van de Walle 2001). Before students are able to develop these relationships, they must be able to count up from 1 to 10 and count back from 10 to 1. To check that students can do this, start with a number less than 10. For example, start with the number 4, and have the students count up to 9. Or start with the number 8, and have the students count back to 5.

The one and two more, one and two less than relationships involve more than the ability to count on or count back one or two. They involve the understanding that, for example, the number 8 is one more than the number 7 and two more than the number 6. Completing addition facts with an addend of 1 or 2 involves the one-more-than and two-more-than relationships.

Check that your students have developed these relationships: Present 6 counters to the students. Ask the students:

- How many counters are there? (6)
- How many counters would there be if 2 counters were added? (6 counters and 2 counters are 8 counters)

Ask the students to explain how they found the answer 8. Their answers will provide you with important information about their number sense development. Some students will say they counted on from the number 6 ("6 → 7, 8"). Others will say they counted all ("1, 2, 3, 4, 5, 6, 7, 8"). Others will say they "just knew" that 8 is two more than the number 6. The students who say they knew that two more than the number 6 is 8 have already developed the two-more-than relationship.

It is important that all students develop the one-more-than and two-more-than relationships. Provide students who are still counting all (e.g., "1, 2, 3, 4, 5, 6, 7, 8") with more experiences with numbers until they are able to count on (e.g., "6 → 7, 8"). The following activities can help students who are counting on to develop the one-more-than and two-more-than relationships. The two examples will help students develop the one-more-than and two-more-than relationships.

(1) Distribute counters to the students. Present five to seven dot-pattern cards to them (page 230). Have them use counters to construct sets that are either one more than or two more than the set shown on the cards.

(2) Play Dot-Pattern Flash (with a twist). Instead of writing the number of dots they see on their response boards, have them write the number that is one more than or two more than the number of dots they see.

Anchoring Numbers to 5 and 10

Our number system is a base ten number system. Because two 5s make ten, the numbers 5 and 10 can serve as anchors for number relationships. For example:

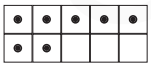
- Students can anchor the numbers 6 and 7 to the number 5. They can think of the number 6 as 5 and 1, and the number 7 as 5 and 2.
- Students can anchor the number 4 to the number 5. They can think of 4 as 5 less 1.
- Students can anchor the numbers 8 and 9 to the number 10. They can think of 9 as 10 less 1, and 8 as 10 less 2.

These relationships are very important and will help the students in completing their addition facts.

Introducing the Ten-Frame

The most powerful model for anchoring the numbers to 5 and 10 is the ten-frame. The ten-frame is an array of 2 rows and 5 columns in which counters or dots are placed to illustrate numbers.

Give each student a ten-frame mat and counters (page 218). Ask the students to use the counters to display the number 7 on their ten-frame. Have them fill the top row first, beginning on the left. Once the first row is full, have them place the remaining counters in the second row, again beginning on the left. Have the students note that 7 is displayed as 5 and 2.



Give the students other numbers from 1 to 10 to display on their ten-frame mats. Encourage the students to note how these numbers are anchored to 5 or 10. For example, 6 is displayed as 5 and 1, and 9 is displayed as 10 less 1.

Have the students make the connection between the ten-frame and their fingers. They have 5 fingers on each hand and 10 fingers on both their hands.

The ten-frame is a powerful model because it allows the students to visually anchor the numbers from 1 to 10 to the numbers 5 and 10 – students can picture the numbers they are adding. The more senses involved in learning, the more effective the learning will be. The ten-frame is used throughout this program.

The Ten-Frame Train

To help the students with the ten-frame, draw a train car with the ten-frame as the window. Explain to the students that this train is filled when there are 10 passengers in it. Have the passengers on the train sit in the order in which counters are placed on ten-frames.

Note: Illustrations of ten-frame trains and passengers are on page 223.

Ten-Frame Flash

Refer to the page containing ten-frames (page 220). Place one ten-frame on the overhead for a few seconds. Turn off the overhead, and ask the students how many dots they saw. Have the students write their answer on their response boards. Check whether the students are able to answer without counting the dots. Continue to do this activity with the students until they can answer without counting the dots.

Part-Part-Whole Relationships

Conceptualizing a number as being made up of two or more parts is the most important relationship a child can develop about numbers (John Van de Walle 2001). The part-part-whole relationship is fundamental to understanding addition.

Check that students understand that 4 and 1 makes 5. The models used in the program reinforce the part-part-whole relationship. This relationship can also be referred to as the part-part-all relationship.

If students need more practice with this relationship, have them create sets with two shapes of pattern blocks or two-colour counters and find the total number in each set.

Thinking Strategy: Addend of 1

The thinking strategy for an addition fact with an addend of 1 is based on the one-more-than relationship.

Consider, for example, the fact 8 + 1.

8 + 1 (addend of 1)

- One-More-Than:

  One more than 8 is 9.
  SO, 8 + 1 = 9

Thinking Strategy: Addend of 2

The thinking strategy for an addition fact with an addend of 2 is based on the two-more-than relationship.

Consider, for example, the fact 5 + 2.

5 + 2 (addend of 2)

- Two-More-Than:

  Two more than 5 is 7.
  SO, 5 + 2 = 7

Check that students can count on 2 correctly.

Thinking Strategy: Addend of 3

The thinking strategy for an addition fact with an addend of 3 is the following:

Consider, for example, the fact 6 + 3.

6 + 3 (addend of 3)

- Count on 3:

  6 → 7, 8, 9
  SO, 6 + 3 = 9

Most students will need to count on 3 from 6. Check that they are able to count on 3.

The Commutative Property of Addition

The commutative, or turnaround, property of addition means that the sum of an addition fact is the same regardless of the order of the numbers being added. It is a powerful tool in mastering addition facts, and it is used throughout the program.

Students are introduced to turnaround facts, for example, 9 + 1 and 1 + 9, 5 + 2 and 2 + 5, 6 + 3 and 3 + 6 together. Since turnaround facts have the same sum, students can use the same strategy for the fact 1 + 9 as they use for 9 + 1, the same strategy for the fact 2 + 5 as they use for 5 + 2, and the same strategy for the fact 3 + 6 as they use for 6 + 3.

Consider, for example, the fact 1 + 9.

1 + 9 (addend of 1)

- One-More-Than:

  One more than 9 is 10.
  SO, 1 + 9 = 10

Thinking Strategy: Addend of 0

The thinking strategy for an addition fact with an addend of 0 is the following:

Consider, for example, the fact 6 + 0.

6 + 0 (addend of 0)

- Pattern of 0:

  The sum of a number and 0 is the number.
  SO, 6 + 0 = 6

The same strategy is used to complete the turnaround fact 0 + 6.

Level 1 consists of the following lessons:

Lesson 1A: Domino Rally with 1, 2, and 3

This lesson introduces the spatial and part-part-whole relationships for the numbers 1 through 10 as well as the operation of addition. The terms *sum* and *addend* are also introduced.

Lesson 1B: Ten-Frame Rally

This lesson introduces the ten-frame model. The ten-frame model is emphasized throughout the program.

Lesson 1C: Ten-Frame Rally with 1, 2, and 3

This lesson introduces the one-more-than and two-more-than relationships for numbers 1 through 10. Thinking strategies for facts with addends of 1, 2, and 3 are also introduced.

Lesson 1D: Match-Ups with 1, 2, and 3

This lesson introduces thinking strategies for facts with addends of 1, 2, and 3.

Lesson 1E: Cube Train Rally with 1, 2, and 3

This lesson introduces the commutative, or turnaround, property of addition.

Lesson 1F: Wacky Webs with 1, 2, and 3

In this lesson, students practice completing facts with an addend of 1, 2, or 3. They are also introduced to even and odd numbers.

Lesson 1G: Domino Rally with 0

This lesson introduces addition facts with an addend of 0.

Lesson 1H: Number Design with 0, 1, 2, and 3

In this lesson, students complete facts with addends of 0, 1, 2, and 3 by identifying and applying the appropriate thinking strategies.

Lesson 1I: Secret Message with 0, 1, 2, and 3

In this lesson, students continue to complete facts with addends of 0, 1, 2, and 3.

Lesson 1J: The Level 1 Number Challenge

This challenge assesses the students' ability to identify and apply appropriate thinking strategies to the facts introduced in Level 1. Students also review models and vocabulary introduced so far in the program.

ADDITION GRID

The addition facts considered in this level are the following:

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	
2	2	3	4	5	6	7	8	9	10		
3	3	4	5	6	7	8	9	10			
4	4	5	6	7							
5	5	6	7	8							
6	6	7	8	9							
7	7	8	9	10							
8	8	9	10								
9	9	10									
10	10										

LESSON 1A: DOMINO RALLY WITH 1, 2, AND 3

TEACHER LESSON

In this lesson, students are introduced to the operation of addition and the part-part-whole relationship. Spatial relationships that students should develop for the numbers that are less than or equal to 10 are also introduced.

Interpreting the Operation of Addition

Present the following story problem to the students:

- Amber has 4 bracelets. Her friend Tania gives her 1 more bracelet. Have many bracelets does Amber have altogether?

Have the students draw a picture showing the story problem and solve the problem.

- Amber has 5 bracelets altogether.

Note: This story problem illustrates the additive structure of joining; result unknown. Other additive structures are possible and are discussed in the Introduction to the program (page 4).

Ask the students to represent the story problem as an addition sentence.

- 4 + 1 = 5 (4 and 1 is 5)

Ask the students to describe other situations that illustrate this fact. Some examples are:

- 4 pencils and 1 pencil are 5 pencils
- 4 apples and 1 apple are 5 apples
- 4 friends and 1 friend are 5 friends

Introductory Terms

Tell the students that in an addition sentence, the parts are called *addends* and the whole is called the *sum*. For example, in the addition sentence 4 + 1 = 5, the numbers 4 and 1 are called the *addends*, and the answer, 5, is called the *sum*.

Part-Part-Whole Relationships

It is important for students to be able to conceptualize a number as being made up of two or more parts. The part-part-whole relationship is fundamental to understanding addition. The models used in the program reinforce the part-part-whole relationship. The domino model reinforces this relationship: the dots on each side of a domino indicate the parts; the total number of dots on the domino indicates the whole. If students need more practice with the development of this relationship, have them create sets with two shapes of pattern blocks or two-colour counters and find the total number in each set.

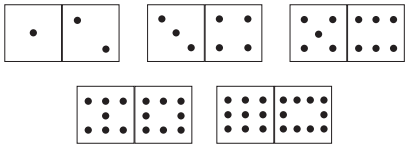
Important! Continue to provide the students with activities that let them practice the part-part-whole relationship until they develop it.

Spatial Relationships

Spatial relationships refer to the ability to recognize sets of 1 to 10 objects in patterned arrangements and tell how many there are without counting. It is one of the important number relationships students should develop for the numbers 1 to 10. Check that your students have developed this relationship by playing games with the Dot Cards (page 230). These can include Fish, Concentration, Snap, and War. The dot cards can also be flashed on the overhead.

The Domino Model

Dominoes provide experiences with recognizing patterned arrangements of dots. The following are the arrangement of 1 to 10 dots on dominoes.



Note: Students will likely find it easier to recognize other arrangement of dots for the numbers 7, 8, and 10 (for example, 4 + 3, 4 + 4, and 5 + 5). Have the students discuss other possible arrangements of dots on dominoes.

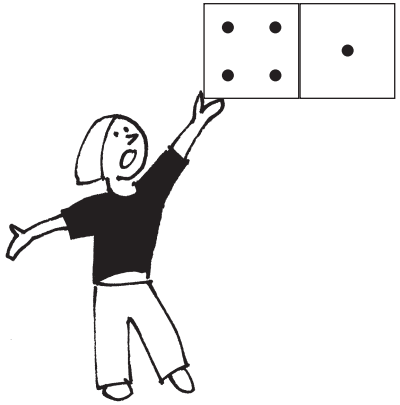
Important! Continue to provide the students with activities that let them practice spatial relationships until they develop them.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Domino Rally, to each student. Read the instructions aloud as a class, and have the students complete the activity. Allow the students to draw their own arrangements of dots on the dominoes.

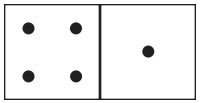


Domino Rally

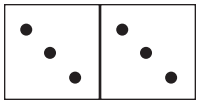
The answer to an addition fact is called the *sum*. The sum of the addition fact 4 + 1 is 5. The addition buddy is showing the addition fact 4 + 1 with a domino.

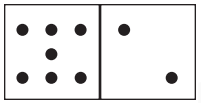
Draw dots on the blank dominoes (below) to show the addition facts, or show what addition fact each domino represents and complete the facts.

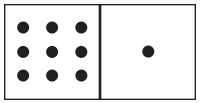
Finally, do what the sentence at the bottom of the page asks.



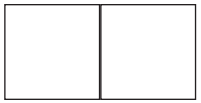
4 + 1 = 5



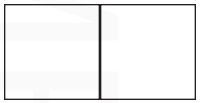




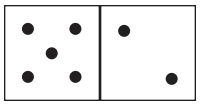
4 + 3 =



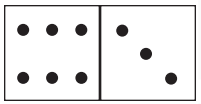
3 + 1 =



8 + 2 =



7 + 1 =



2 + 2 =

Draw dots that show three different dominoes with sums of 8.

LESSON 1B: TEN-FRAME RALLY

TEACHER LESSON

In this lesson, students are introduced to the ten-frame. The ten-frame is perhaps the most important model for anchoring numbers in our number system. Ten-frames are a powerful visual model that can not only help students complete their addition facts but also help students understand the base ten number system. The ten-frame model is emphasized throughout the program.

Anchoring Numbers to 5 and 10

Our number system is a base ten number system. Because two fives make ten, the numbers 5 and 10 can serve as anchors in our numeration system.

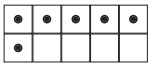
- Students can anchor the numbers 6 and 7 to 5. They can think of the number 6 as 5 and 1, and the number 7 as 5 and 2.
- Students can think of the number 4 as 5 less 1.
- Students can anchor the numbers 8 and 9 to 10 by thinking of 9 as 10 less 1, and 8 as 10 less 2.

These relationships are very important and will help the students in completing their addition facts.

Introducing the Ten-Frame

The most powerful model for anchoring the numbers to 5 and 10 is the ten-frame. The ten-frame is an array of 2 rows and 5 columns in which counters or dots are placed to illustrate numbers.

Give each student counters and a ten-frame mat. Have the students use the counters to display the number 6 on their ten-frame mat.



Explain to students that although there are many ways to place 6 counters on a ten-frame, the counters are to be placed as follows:

- Fill the top row, beginning on the left.
- When the first row is full, place the remaining 1 counter in the second row, again beginning on the left.

Have the students note that 6 is displayed as 5 and 1.

Give the students other numbers from 1 to 10, and have them display these numbers on their ten-frame mats. Encourage the students to note how these numbers are anchored to 5 and 10. For example, 7 is displayed as 5 and 2, and 9 is displayed as 10 less 1.

Point out to the students the connection between the ten-frame and the fingers on their hands. They have 5 fingers on each hand and 10 fingers on both their hands.

The Ten-Frame Train

To reinforce the ten-frame, draw a train with a ten-frame as the window. Have the students place passengers on the train in the same order as they place counters on a ten-frame.

Note: Illustrations of ten-frame trains and passengers are on page 223.

Ten-Frame Flash

Refer to the page of ten-frames (page 220). Flash a ten-frame on the overhead for a few seconds. Ask the students to write the number of dots they saw on the ten-frame on their response boards.

When the students have finished writing the number on their response boards, show them the ten-frame. Explain to the students how the number of dots compares to the number 5 or 10. For example, if there are 6 dots on the ten-frame, point out there are 5 dots in the first row and 1 dot in the second row, and that 1 more than 5 is 6. If there are 7 dots on the ten-frame, point out there are 5 dots in the first row and 2 dots in the second row.

If ten-frames are new to students, consider playing Ten-Frame Flash with ten-frames that have dots in only the first row.

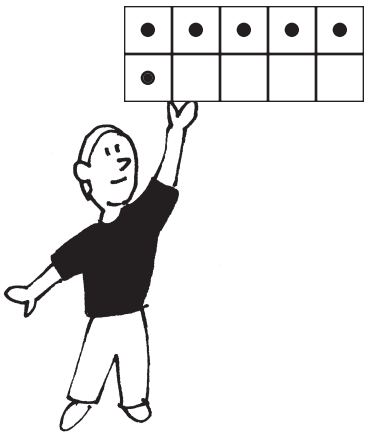
Important! Continue to play Ten-Frame Flash with the students until they can recognize the number of dots on the ten-frames without counting.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

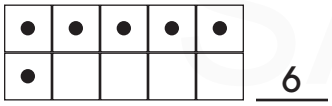
Distribute a copy of the sheet, Ten-Frame Rally, to each student. Read the instructions aloud as a class, and have the students complete the activity. (The students must show each of the numbers from 1 through 10 on the given ten-frames.)

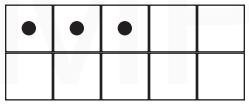


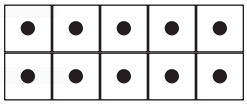
Ten-Frame Rally

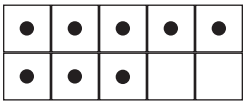
The addition buddy is showing the number 6 with a ten-frame that has 5 dots in the first row and 1 dot in the second row.

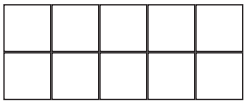
Below, use the ten-frames to show each of the numbers from 1 up to and including 10. All of the ten-frames must have a different number of dots. In some of the frames, you will have to fill in the missing dots.

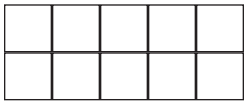
 6

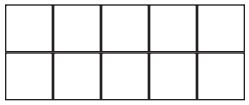
 3

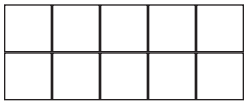
 ___

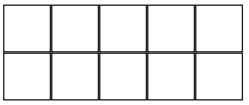
 ___

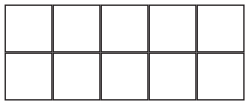
 5

 9

 ___

 ___

 ___

 ___

LESSON 1C: TEN-FRAME RALLY WITH 1, 2, AND 3

TEACHER LESSON

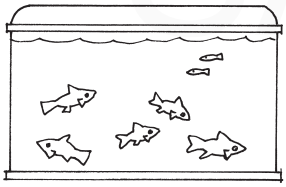
In this lesson, students are introduced to the one-more-than and two-more-than relationships. They also model addition facts with addends of 1, 2, and 3 with ten-frames.

Interpreting the Operation of Addition

Present the following story problem to the students:

- Tyler has 5 goldfish and 2 neons in his fish tank. How many fish does he have altogether in his fish tank?

Have the students illustrate this story problem and solve it.



- Tyler has 7 fish altogether.

Note: This story problem illustrates the additive structure of part-part-whole; whole unknown. Other additive structures are possible and are discussed in the Introduction to the program.

Ask the students to represent the story problem as an addition sentence.

- 5 + 2 = 7

The One-More-Than and Two-More-Than Relationships

The one-more-than and two-more-than relationships are important relationships that students should develop for the numbers 1 through 10.

Present the students with the addition facts 5 + 2 and 8 + 1, and have them complete the facts.

5 + 2 = 7
8 + 1 = 9

Check how the students have completed these facts. It is important that all students develop the one-more-than and two-more-than relationships. Provide students who are still counting all (e.g., "1, 2, 3, 4, 5, 6, 7, 8") with more experiences with numbers until they are able to count on (e.g., "6 → 7, 8"). The following activities can help students who are counting on to develop the one-more-than and two-more-than relationships. Students who complete the facts because they, for example, "just know that 2 more than the number 5 is 7" have developed the two-more-than relationship.

See the Level 1 Overview for activities that will help students develop the one-more-than and two-more-than relationships (page 5).

Important! Continue to provide the students with experiences to practice the one-more-than and two-more-than relationships until they develop them.

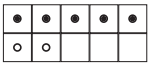
Modelling Addition Facts with Ten-Frames

Give each student counters of two colours and a ten-frame mat. Present the students with the following addition facts:

5 + 2
8 + 1
7 + 3

Have the students model the facts on their ten-frame, representing the two addends with different coloured counters. Have them complete the facts.

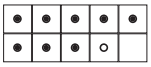
- The addition fact 5 + 2 is modelled with 5 counters in the first row and 2 counters in the second row.



Point out to the students that 5 counters in the first row and 2 counters in the second row show the number 7.

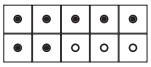
Note: The ten-frame encourages students to recognize that the number 7 is two more than the number 5.

- The addition fact 8 + 1 is modelled as follows:



Point out to the students that 8 counters and 1 counter leave one empty square in the ten-frame.

- The addition fact 7 + 3 is modelled as follows:



Point out to the students that 7 counters and 3 counters fill the ten-frame.

Ten-Frame Flash (with a twist)

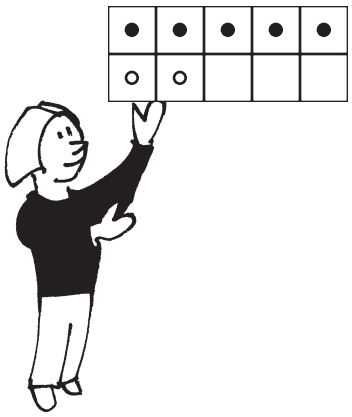
When students can recognize the dots on a ten-frame, play Ten-Frame Flash (with a twist). Show them ten-frames, and ask them for the number that is one-more-than or two-more-than the number indicated on the ten-frame.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

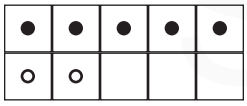
Distribute a copy of the sheet, Ten-Frame Rally, to each student. Read the instructions aloud as a class, and have the students complete the activity.



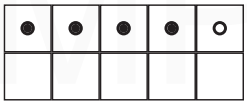
Ten-Frame Rally

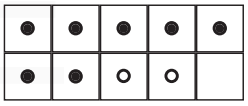
The addition buddy is showing the addition sentence 5 + 2 = 7 with a ten-frame that has 5 solid dots in the first row and 2 dots in the second row.

Name the addition sentences shown by the ten-frames or show the addition facts on the ten-frames. Complete the facts. Then do what the sentences at the bottom of the page ask.

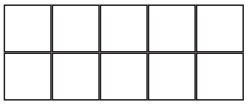


5 + 2 = 7

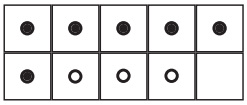


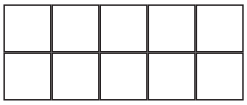


3 + 3 =

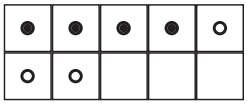


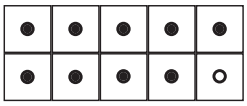
8 + 2 =





5 + 1 =





7 + 3 =

Show the thinking fact 5 + 3 on a ten-frame. Complete the fact. Explain your answer.

LESSON 1D: MATCH-UPS WITH 1, 2, AND 3

TEACHER LESSON

In this lesson, students are introduced to thinking strategies for facts with addends of 1, 2, and 3.

Introducing the Thinking Strategies: Addend of 1 or 2

The one-more-than and two-more-than relationships are the bases of the thinking strategies for facts with an addend of either 1 or 2. Present students with the following facts:

9 + 1
7 + 2

Have the students use the one-more-than and two-more-than relationships to complete the facts.

(a) 9 + 1 (addend of 1)

- One-More-Than:
  One more than 9 is 10.
  SO, 9 + 1 = 10

(b) 7 + 2 (addend of 2)

- Two-More-Than:
  Two more than 7 is 9.
  SO, 7 + 2 = 9

Counting on 1 or 2

If students have not yet developed the one-more-than and two-more-than relationships, encourage them to complete facts 9 + 1 and 7 + 2 as follows.

For 9 + 1, have them count on 1 from 9: "9 → 10."
To complete the fact 7 + 2, have them count on 2 from 7: "7 → 8, 9."

Note: If students have not developed the one-more-than and two-more-than relationships, provide them with experiences until they develop them. Examples of activities that will help them develop these relationships are given in the Level 1 Overview.

Introducing the Thinking Strategy: Addend of 3

Counting on 3 is the basis of the thinking strategy for facts with an addend of 3. Present the following fact to the students.

5 + 3 (addend of 3)

- Count on 3:
  5 → 6, 7, 8
  SO, 5 + 3 = 8

Counting on 3

Encourage the students to count on 3 as follows: Have them say the number 5, pause, then say the numbers 6, 7, 8. Have them say these numbers in a rhythm.

Show them how to count on 3 a few times (e.g., for 5 + 3, say: "5 → 6, 7, 8"), then have them count on together out loud as a class. You can also clap your hands as they count on.

Next, present a number from 3 to 10, and have the students mouth the numbers as you clap your hands.

Continue to do this throughout the first level until students can successfully count on 3.

Note: Encourage students who are still using their fingers to count on without their fingers.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Match-Ups, to each student. Read the instructions aloud as a class, and hve the students complete the activity.

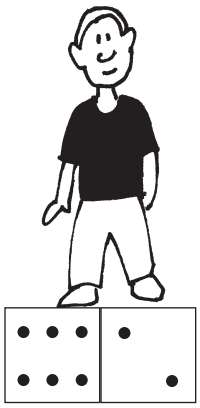
Note: Students will find that the connecting lines of each set of addition facts form a pattern. They can use the patterns to check their work.

Note: If students need help completing the facts in this activity, have them model the facts on the blank ten-frames (page 219). Continue to have them do so until they are able to use the thinking strategies to complete the facts.

COMPLETING THE ADDITION GRID

Distribute a copy of the addition grid (page 132) to each student. Tell students they will be using this grid throughout the program. Explain to them that the grid will provide them with a record of their progress.

Have the students fill in the addition grid for all facts with addends of 1, 2, and 3, and with sums that are less than or equal to 10. Have students fill in both rows and columns of the grid.



Match-Ups

Complete each addition fact by connecting the dot beside the fact to the dot beside its sum.

When you have completed the "match-ups," answer the question at the bottom of the page.

6 + 2 •	• 4	5 + 3 •	• 4
7 + 3 •	• 9	2 + 2 •	• 8
8 + 1 •	• 10	9 + 1 •	• 5
2 + 2 •	• 8	5 + 2 •	• 7
4 + 3 •	• 5	4 + 1 •	• 10
1 + 1 •	• 2	6 + 3 •	• 2
2 + 3 •	• 7	2 + 1 •	• 6
4 + 2 •	• 3	3 + 3 •	• 3
1 + 2 •	• 6	1 + 1 •	• 9

What thinking strategy did you use for completing the fact 6 + 2? Explain your answer.

LESSON 1E: CUBE TRAIN RALLY WITH 1, 2, AND 3

TEACHER LESSON

In this lesson, students are introduced to pairs of facts like 6 + 2 and 2 + 6 at the same time and will examine whether or not turnaround facts have the same sum.

Turnaround Facts

Present the addition fact 6 + 2 to the students. Beside it, write the addition fact 2 + 6. Tell the students that these two facts can be called *turnaround facts*. Divide the class into cooperative working pairs. Distribute a copy of the student activity sheet to each pair of students.

Modelling Facts

Distribute two colours of interlocking cubes to each pair of students, and have them use these cubes to construct cube trains. Have them use one colour to represent one addend and the other colour to represent the other addend. Have one student in each pair model the fact 6 + 2 and the other student model its turnaround, 2 + 6. The cube trains for the facts 6 + 2 and 2 + 6 are modelled as follows:

6 + 2 2 + 6

The Commutative Property of Addition

After all students have completed the activity sheet, ask them: Do turnaround facts have the same sum?

- Turnaround facts have the same sum. The order of the addends does not affect the sum.

Have them use their cube trains to explain why turnaround facts have the same sum. If they have difficulty, offer the following explanation:

- Consider the cube train modelling the fact 6 + 2. If the cube train is turned and viewed from right to left, or turned 180°, it now looks like the cube train modelling the fact 2 + 6. Both cube trains are made up of the same number of cubes. The sum of 6 + 2 is the same as the sum of 2 + 6. This explanation is true for all turnaround facts.

Explain to students that this property is called the *turnaround* or *commutative property of addition*. The commutative property of addition means that the sum of an addition fact is the same regardless of the order of its addends. It is a powerful tool in mastering the addition facts, and it is used throughout the program.

Identifying Turnaround Facts

Present students with pairs of facts, some that are turnarounds and some that are not. Have the students indicate the pairs of facts that are turnarounds by either raising their hands or giving the thumbs-up sign.

Applying the Commutative Property of Addition

Present the addition fact 2 + 6 to the students. Have them complete the fact and explain their thinking strategy.

2 + 6 (addend of 2)

- Two-More-Than:

  Two more than 6 is 8.
  SO, 2 + 6 = 8

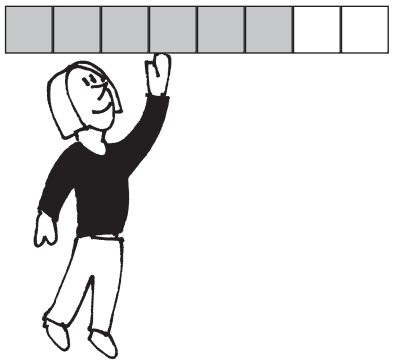
The commutative property of addition allows the students to use the two-more-than strategy whether or not 2 is the first or the second addend of the fact. When students are asked to complete a fact with an addend of 1, 2, or 3, have them begin with the greater number and find one, two, or three more than the larger number. For example, for the fact 2 + 6, if students have not yet developed the two-more-than relationship, have them start with the larger number, 6, then pause, and count on 7, 8. Present other facts that have a first addend of 1, 2, or 3, and have the students complete them. Ask the students which number they should begin with (the greater number). Have them complete the fact.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Students will have completed the activity sheet and explained why the pair of turnaround facts 6 + 2 and 2 + 6 have the same sum.



Cube Train Rally

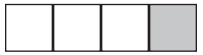
You can show the addition facts 6 + 2 and 2 + 6 with cube trains.

The numbers 6 and 2 are called *addends*, and the facts 6 + 2 and 6 + 3 are called *turnarounds.*

Below, name the addition sentences shown by the cube trains, and find the sum of each.
Then, answer the question at the bottom of the page.

6 + 2 = 8

2 + 6 = 8



Do the addition facts 6 + 2 and 2 + 6 have the same sum? Explain your answer.

LESSON 1F: WACKY WEBS WITH 1, 2, AND 3

TEACHER LESSON

In this lesson, students practice completing addition facts with an addend of 1, 2, or 3, and they are introduced to even and odd numbers.

Identifying and Applying Thinking Strategies

Present students with the following addition facts:

3 + 6
2 + 7
1 + 9

Have students complete each fact and explain the thinking strategy they used for each. Have the students first note whether the facts have an addend of 1, 2, or 3.

(a) 3 + 6 (addend of 3)

- Count on 3:
  6 → 7, 8 ,9
  SO, 6 + 3 = 9

Note: Make sure the students recognize that the fact has an addend of 3; they can start with 6 and count on 3.

(b) 2 + 7 (addend of 2)

- Two-More-Than:
  Two more than 7 is 9.
  SO, 2 + 7 = 9

Note: Check that students recognize the fact has an addend of 2; they can start with 7 and use the two-more-than relationship.

(c) 1 + 9 (addend of 1)

- One-More-Than:
  One more than 9 is 10.
  SO, 1 + 9 = 10

Note: Check that students recognize the fact has an addend of 1; they can start with 9 and use the one-more-than relationship.

Practicing One-More-Than, Two-More-Than, and Count on 3

To provide practice in completing facts with addends of 1, 2, and 3, consider the following:

Present numbers from 1 to 9, and ask the students to name the number that is one more than each number. Next, present numbers from 1 to 8, and ask the students to name the number that is two more than each number. Do the same for the numbers from 1 to 7; ask the students to name the number that is three more than each number.

Even and Odd Numbers

Distribute 10 cubes to each student. Ask the students to divide the cubes into 2 equal groups.

- The 10 cubes can be divided into 2 groups of 5.

Explain to the students that a number such as 10 (which represents 10 objects) that can be divided into 2 equal groups is called an *even number*.

Have the students use the cubes to find other even numbers less than or equal to 10.

- The numbers 2, 4, 6, and 8 are also even numbers because they can be divided into 2 equal groups.

Tell the students that the number 0 is also considered to be an even number.

Now ask the students if they can divide 9 cubes into 2 equal groups.

- The 9 cubes cannot be divided into equal groups.

Tell the students that a number like 9 (which represents 9 objects) that cannot be divided into 2 equal groups is called an *odd number*.

Have the students use the cubes to find other odd numbers less than or equal to 10.

- The numbers 1, 3, 5, 7, and 9 are also odd numbers because they cannot be divided into 2 equal groups.

Note: Further discussion of even and odd numbers, including rules for even and odd sums, is given in Lesson 6G. This lesson only introduces students to even and odd numbers.

Encouraging Class Discussion

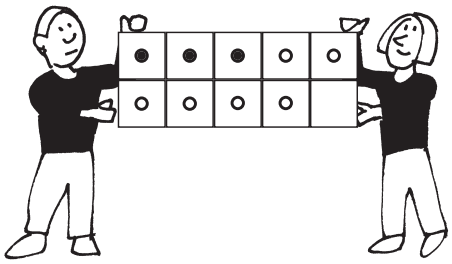
Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Wacky Webs, to each student. Read the instructions aloud as a class, and have the students complete the activity. (In addition to completing the facts, students have to colour all cells – middle, inner, and outer – that have an even number.)

Note: The coloured cells form a pattern that students can use to check their work. The pattern provides an introduction to the rules for even and odd sums.

Wacky Webs

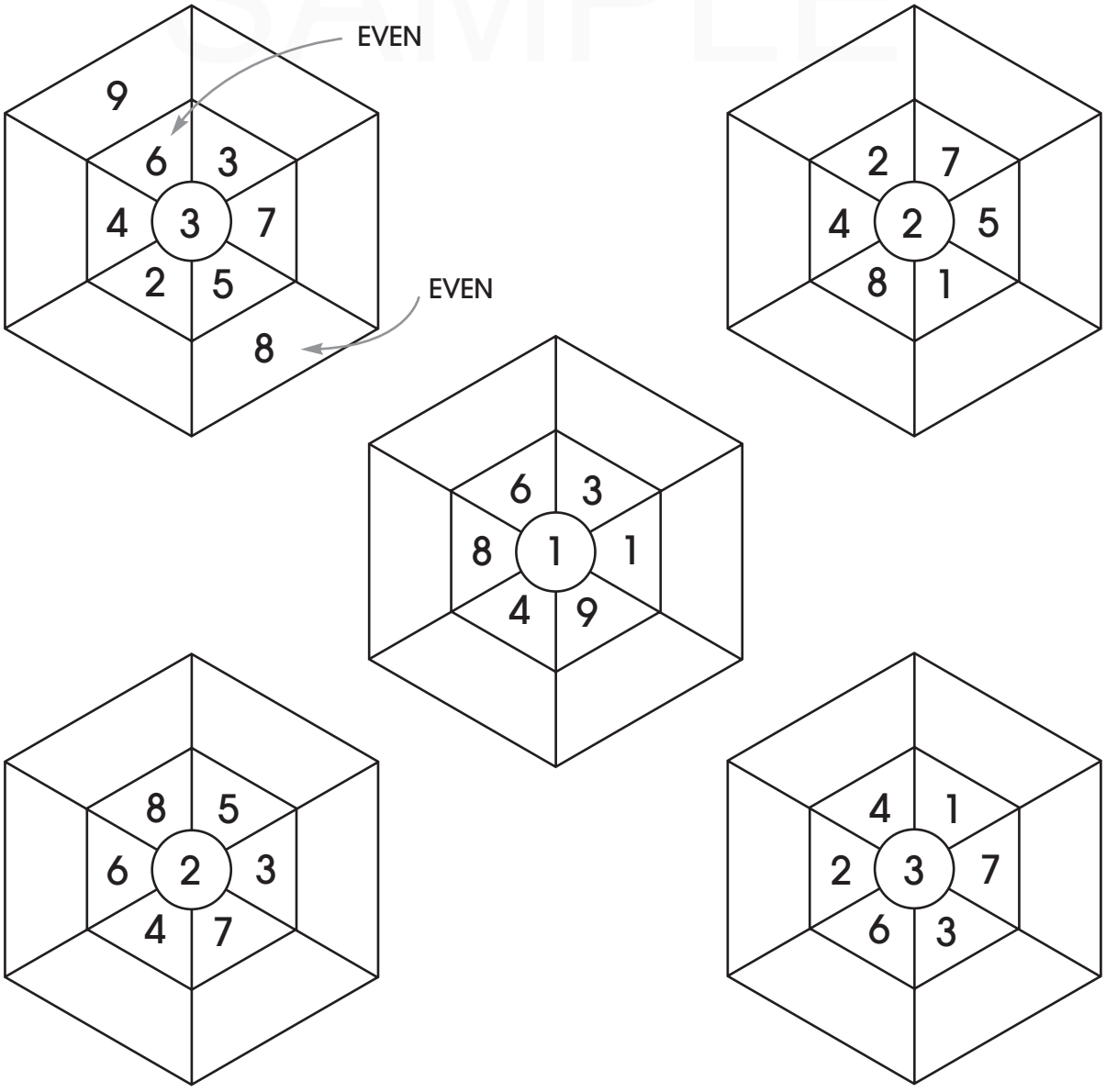


The numbers 0, 2, 4, 6, 8, and 10 are **EVEN** numbers.

The numbers 1, 3, 5, 7, and 9 are **ODD** numbers.

Below, fill in the outer cells of each web by adding the numbers in the inner cells with the number in the middle.

Colour all of the cells that have an **EVEN** number.



LESSON 1G: DOMINO RALLY WITH 0

TEACHER LESSON

In this lesson, the students are introduced to addition facts with an addend of 0.

The Concept of the Number 0

Ask the students what the number 0 means to them. The concept of a number 0 is not an easy one. Discussing the number 0 as a class can be helpful.

Interpreting the Operation of Addition

Present the addition fact 5 + 0 to the students. Have them illustrate the fact. Examples can include the following:

- 5 pencils and 0 pencils are 5 pencils
- 5 candies and 0 candies are 5 candies
- 5 dogs and 0 dogs are 5 dogs

Ensure students understand that when they add 0 to a number, the sum is that number. Some students may believe that when two numbers are added, the sum is always greater than the numbers being added. Have them note that when 0 is added to a number, nothing is being added to the number, and the sum is just the number.

Distribute a copy of the student activity sheet to each student. Divide the class into cooperative working pairs. Have the students model the facts on the activity sheet by drawing dots on the dominoes and/or by naming the addition sentences the dominoes show. Each domino on the activity sheet must have a different sum, and one side must remain blank. Have the students explain their thinking strategy for adding a fact with an addend of 0.

Creating Story Problems

It is important that students understand why the sum of a number and 0 is the number, not simply be told that it is a rule they must follow. One way to help the students understand what it means to add with 0 is to have them create story problems.

Ask the students to create story problems illustrating the fact 5 + 0. Suggest the story problem involve crackers. Have them solve their story problems and draw pictures illustrating them.

Introducing the Thinking Strategy: Addend of 0

When students have completed the student activity sheet, ask them what thinking strategy they used for completing a fact with an addend of 0.

- The sum of a number and 0 is the number.

Note: Check that students understand the strategy and are able to complete facts with 0. If students require practice with facts with an addend of 0, have them work with dominoes or other manipulatives until they understand the strategy. They can also model these facts on blank ten-frames (page 219).

Practicing the Thinking Strategy: Addend of 0

Present the following addition facts to the students:

5 + 0
0 + 7

Have students complete these facts with the strategy for facts with an addend of 0.

(a) 5 + 0 (addend of 0)

- Pattern of 0:

  The sum of a number and 0 is the number.
  SO, 5 + 0 = 5

(b) 0 + 7 (addend of 0)

- Pattern of 0:

  The sum of 0 and a number is the number.
  SO, 0 + 7 = 7

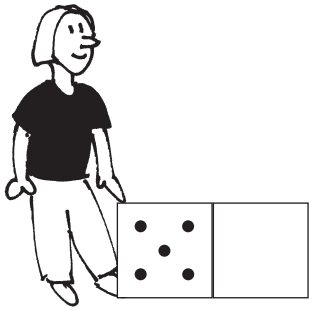
Note: Ensure students recognize they can use the pattern of 0 strategy regardless of whether 0 is the first addend or the second addend of the facts.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

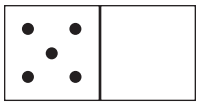
Students will have completed the activity sheet and explained the thinking strategy for addition facts with an addend of 0.



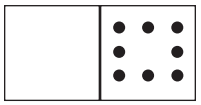
Domino Rally

The addition buddy is standing beside a domino that shows the addition fact 5 + 0.

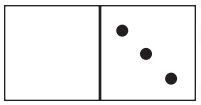
Below, fill in the dominoes, and complete the addition fact that each shows.
All dominoes must have one side with no dots, and each must have a different sum.
Then, answer the question at the bottom of the page.



5 + 0 = _____

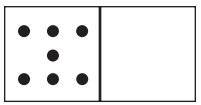


_____



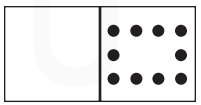
_____

1 + 0 = _____



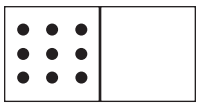
_____

0 + 4 = _____



_____

0 + 0 = _____



_____

_____

_____

What thinking strategy do you use when you add a number with 0? Explain your answer.

LESSON 1H: NUMBER DESIGN WITH 0, 1, 2, AND 3

TEACHER LESSON

In this lesson, students complete facts with addends of 0, 1, 2, and 3 by identifying and applying the appropriate thinking strategies.

Identifying and Applying Thinking Strategies

Present the students with the following addition facts, each with a missing addend:

	3	0			2
+ 6	+ ___	+ ___	+ 7	+ 5	+ ___

Have the students replace the missing addend with the number 0.

0	3	0	0	0	2
+ 6	+ 0	+ 0	+ 7	+ 5	+ 0

Have the students complete the facts.

0	3	0	0	0	0
+ 6	+ 0	+ 0	+ 7	+ 5	+ 2
6	3	0	7	5	2

Have students explain their thinking strategy for completing the facts: The facts can all be completed using the pattern of 0. Remind the students that this strategy can be used for all facts with an addend of 0. Consider, for example, the fact 0 + 6.

0 + 6 (addend of 0)

- Pattern of 0:

  The sum of 0 and a number is the number.
  SO, 0 + 6 = 6

Next have the students replace the missing addends with the number 1. Have them complete the facts and explain their thinking strategy.

Have students continue by replacing the missing addends by the number 2 and then the number 3.

Practicing Identifying Facts

If students need practice in identifying the facts, do the following:

1. Give each student four pieces of paper.
2. Have the students write one of the numbers “0,” “1,” “2,” and “3” on each piece.
3. Present addition facts to the students.

Ask students to indicate if the facts are pattern of 0, one-more-than, two-more-than, or count on 3 facts by holding up the piece of paper with that number on it. If a fact does not have an addend of 0, 1, 2, or 3, tell them not to hold up any paper.

Addition Facts with Ten-Frames

For students who are having difficulty completing facts with addends of 0, 1, 2, and 3, have them model the facts on the activity sheet of this lesson on the blank ten-frames found on page 219.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Number Design, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Note: The coloured areas form a pattern that students can use to check their work.

CHALLENGE FACTS

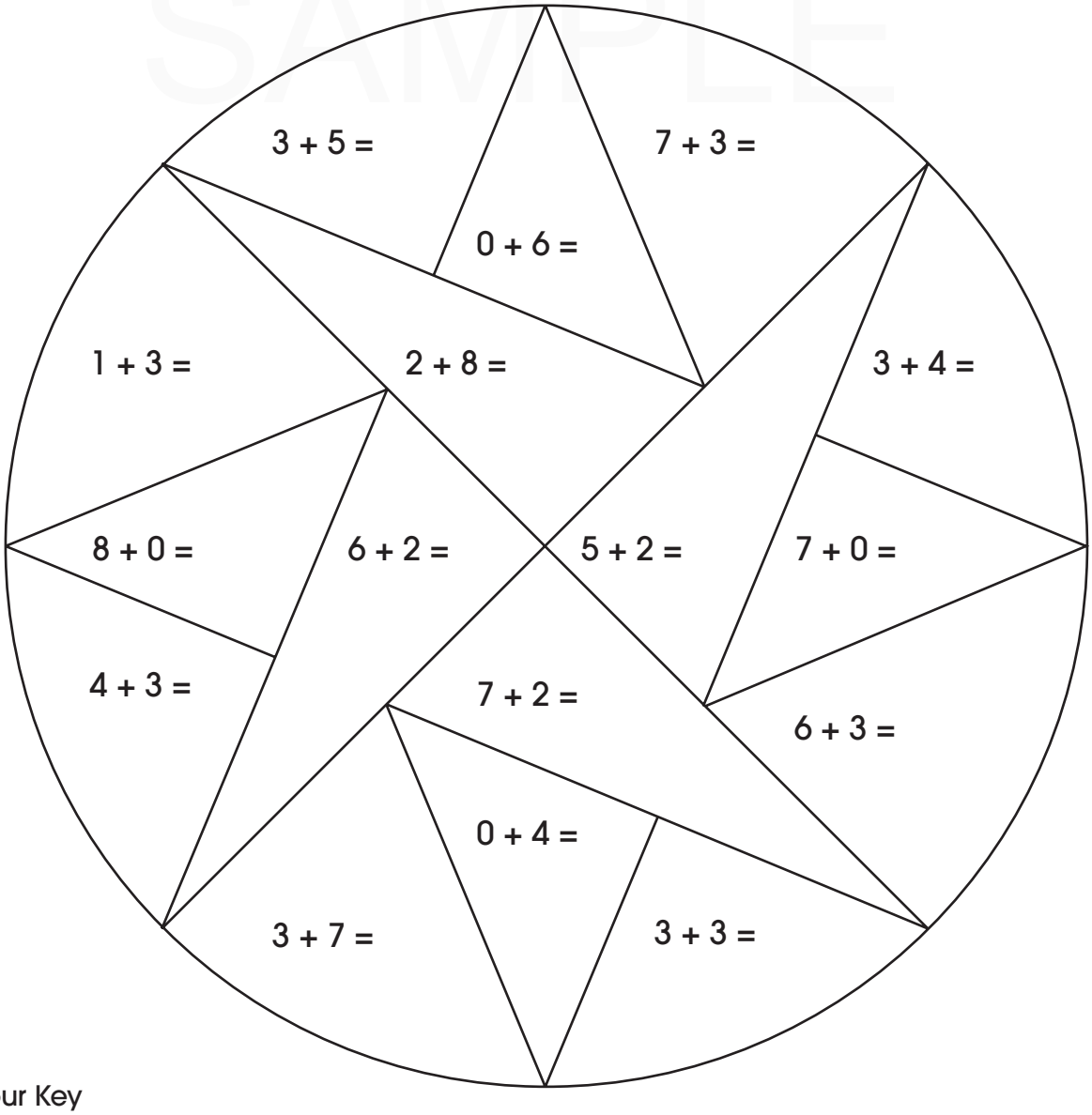
Use the Challenge Facts, Level 1 (pages 180-181), to help the students recognize facts with addends of 0, 1, 2, and 3. Have students place shapes around the facts with the same addends (for example, circles around all the facts with an addend of 2) and then complete them. Next, have them place triangles around the facts with an addend of 3, and so on. They can also use different coloured pencils to identify the facts.

POWER FACTS

Have students use Power Facts to practice the thinking strategies for facts with an addend of 0, 1, 2, or 3. Hand out the Power Facts, Level 1 (page 133), and have the students practice the first set of Power Facts at least once a day. Students can practice in class or at home. At this point in the program, also give students the letter for parents (page 146) to take home.

Number Design

Complete the addition facts below. When you are finished, follow the Colour Key, and colour the design.



Colour Key

addend of 0: green

addend of 2: blue

addend of 3: yellow

LESSON 11: SECRET MESSAGE WITH 0, 1, 2, AND 3

TEACHER LESSON

In this lesson, students complete facts with addends of 0, 1, 2, and 3.

Practicing Thinking Strategies

Present the following addition facts to the students:

3 + 7
0 + 8
9 + 1
2 + 3

Have the students complete the facts. The students must be able to recognize and apply an appropriate thinking strategy for each fact, then explain their thinking strategy. Have the students first note whether the facts have an addend of 0, 1, 2, or 3.

(a) 3 + 7 (addend of 3)

- Count on 3:
  7 → 8, 9, 10
  SO, 3 + 7 = 10

Note: Some students will complete this fact by picturing the ten-frame. Encourage them to do so.

(b) 0 + 8 (addend of 0)

- Pattern of 0:
  The sum of 0 and a number is the number.
  SO, 0 + 8 = 8

(c) 9 + 1 (addend of 1)

- One-More-Than:
  One more than 9 is 10.
  SO, 9 + 1 = 10

Note: Some students will complete this fact by picturing the ten-frame. Encourage them to do so.

(d) 2 + 3 (addend of 2/addend of 3)

- Two-More-Than:
  Two more than 3 is 5.
  SO, 2 + 3 = 5
- Count on 3:
  2 → 3, 4, 5
  SO, 2 + 3 = 5

Note: Because the fact 2 + 3 has both an addend of 2 and an addend of 3, students can choose a strategy for either addend. Discuss with students why it is easier to start with 3 and use the two-more-than strategy.

Continue to ask the students to complete addition facts with addends of 0, 1, 2, and 3.

Ten-Frame Flash

Continue to play Ten-Frame Flash with the students. In addition to recognizing the number shown on the ten-frame, ask them for the number that is one-more-than or two-more-than the number shown.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Secret Message, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Note: Take time to explain how the blanks are completed in this Secret Message. All the Secret Messages in the program are completed in this way. If students have difficulty in completing the code, have them write the letter that matches each sum beside each fact. Then have them place these letters in the blanks one after the other. Some students like to keep a finger on the fact; others like to check off each fact as they place its matching letter in the Secret Message.

PARTNER BINGO

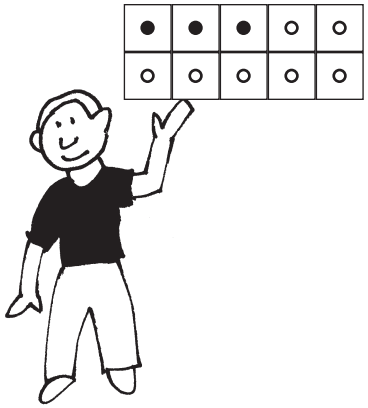
Students can practice the addition facts with addends of 0, 1, 2, and 3 by playing Partner Bingo, Level 1 (pages 148-150). Have the students complete the facts in order and cross out only one square on their cards for each fact. Partner Bingo can be played in class or at home.

CARD GAMES

Card Games, Level 1, provide more practice for the facts with addends of 0, 1, 2, and 3 (pages 204-205). Card Games can be played in class or at home.

STUDENT JOURNAL

Have students choose four facts: one with an addend of 0, one with an addend of 1, one with an addend of 2, and one with an addend of 3. Have them complete the facts and explain the thinking strategies they used.



Secret Message

Complete the addition facts. The first fact is completed: 3 + 7 = 10. Check the Code Key. The letter above 10 is A. The letter A is written in the first blank. The second fact is completed: 0 + 1 = 1. Check the Code Key. The letter above 1 is T. The letter T is written in the second blank. Continue with each addition fact until you have filled in the remaining blanks with a letter from the Code Key. When you are finished, you will find the secret message.

Where do acrobats learn to walk the tightrope?

A	T										
①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫

Code Key	C	T	G	L	I	S	O	H	A
	0	1	3	5	6	7	8	9	10

① 3 + 7 = 10	② 0 + 1 = 1	③ 7 + 2 =	④ 1 + 5 =	⑤ 3 + 0 =	⑥ 6 + 3 =
⑦ 5 + 2 =	⑧ 0 + 0 =	⑨ 1 + 8 =	⑩ 2 + 6 =	⑪ 5 + 3 =	⑫ 3 + 2 =

LESSON 1J: THE LEVEL 1 NUMBER CHALLENGE

The Level 1 Number Challenge involves the addition facts introduced in Level 1.

Before students take the Level 1 Number Challenge, review the terms *addend* and *sum*. Have them complete addition facts with addends of 0, 1, 2 and 3. Choose facts that the students find most challenging. The following are often difficult for students to complete:

2 + 7
1 + 9
6 + 3
9 + 0
3 + 7
0 + 1
2 + 8

Have the students discuss the thinking strategies they can use with these addition facts.

INSTRUCTIONS FOR THE LEVEL 1 CHALLENGE

Distribute a copy of the challenge sheet to each student. Explain the challenge: There are nine facts at the top of the page. Below the facts, there are nine boxes with a clue in each box. Complete the facts, then match each completed fact with one of the clues and place it in that box. Cross out each fact after it has been placed in its box. For students who would like an extra challenge, have them add the sums of the three facts in each row, then add the sums of the three facts in each column. (See answer key for the mystery number.)

SUGGESTIONS FOR MORE PRACTICE

Students will have many more opportunities to practice the addition facts of this level as they work through the program. However, if they need more practice, consider the following suggestions:

- Check that students have developed the relationships for the numbers 1 through 10 explained in the overview of Level 1 (pages 5-7). These relationships are prerequisites to building mastery of the addition facts. They are the following:

  1. Spatial Relationships
  2. One-More and Two-More, One-Less and Two-Less
  3. Anchors or benchmarks of 5 and 10
  4. Part-Part-Whole

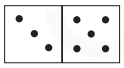
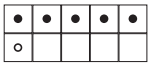
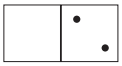
  Continue to provide the students with experiences that develop these relationships. Activities that provide experiences with these relationships are provided in the Overview of Level 1 (pages 5-7).

- Check that students can identify addition facts with an addend of 0, 1, 2, or 3. The addend can either be the first addend or the second.
- Check that students can count on 1, 2, and 3 when they are given a number that is less than 10. Have students practice counting on until they can do so confidently.
- Check that students understand the thinking strategy for addition facts with an addend of 0.
- Identify the facts that students find difficult. Help them develop thinking strategies for these facts. Have them list the facts they find difficult, and encourage them to practice these facts both with a classmate in school and a parent at home.
- Have students continue to practice the first set of Power Facts.
- Have students play Partner Bingo in class or at home. There are 3 Partner Bingos for this level of the program (pages 148-150).

Level 1 Number Challenge

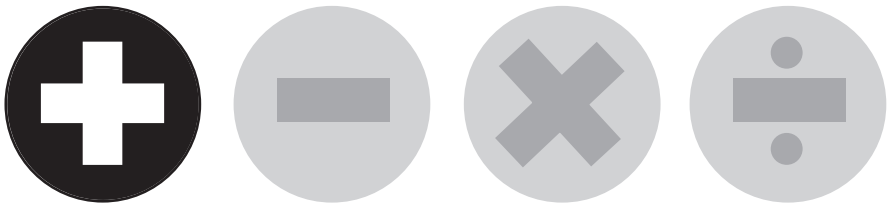
Complete the addition facts below. Then, read the clue in each box, and find the fact that matches it. Place each completed fact into the box with its matching clue.

2	1	2	5	3	0	5	1	3
+ 3	+ 0	+ 2	+ 2	+ 6	+ 2	+ 1	+ 2	+ 5

This fact is shown with the following picture: 	This fact joins 1 and 2.	This fact has the same addends.
This fact has the smallest sum.	This fact is shown with the following picture: 	The sum of this fact is three more than 6.
This fact is shown with the following picture: 	The sum of this fact is two more than 5.	This fact is shown with the following picture: 

APPENDIX A

TEACHER RESOURCES



POWER FACTS: LEVEL 1

9 + 1 = 10	4 + 3 = 7	2 + 6 = 8	9 + 0 = 9
2 + 7 = 9	1 + 8 = 9	3 + 1 = 4	3 + 3 = 6
3 + 5 = 8	0 + 0 = 0	6 + 3 = 9	1 + 7 = 8
0 + 6 = 6	3 + 2 = 5	2 + 8 = 10	5 + 2 = 7
1 + 2 = 3	3 + 7 = 10	4 + 2 = 6	1 + 6 = 7

THINKING STRATEGIES

Level 1 introduces facts with an addend of 0, 1, 2, or 3.

THINKING STRATEGY: ADDEND OF 1

The thinking strategy for an addition fact with an addend of 1 is based on the one-more-than relationship.

Consider, for example, the fact 9 + 1.

9 + 1 (addend of 1)

- One-More-Than:

  One more than 9 is 10.
  SO, 9 + 1 = 10

THINKING STRATEGY: ADDEND OF 2

The thinking strategy for an addition fact with an addend of 2 is based on the two-more-than relationship.

Consider, for example, the fact 2 + 7.

2 + 7 (addend of 2)

- Two-More-Than:

  Two more than 7 is 9.
  SO, 2 + 7 = 9

If students have not yet developed the two-more-than relationship, suggest they count on 2 as follows: Have them start with the greater addend, 7. Have the students say 7, pause, and say 8, 9.

THINKING STRATEGY: ADDEND OF 3

The thinking strategy for an addition fact with an addend of 3 is the count on strategy.

Consider, for example, the fact 3 + 5.

3 + 5 (addend of 3)

- Count on 3:

  5 → 6, 7, 8
  SO, 3 + 5 = 8

Suggest the students count on 3 as follows. Have them start with the greater addend, 5. Have the students say 5, pause, and say 6, 7, 8.

THINKING STRATEGY: ADDEND OF 0

The thinking strategy for an addition fact with an addend of 0 is the pattern of 0.

Consider, for example, the fact 0 + 6.

0 + 6 (addend of 0)

- Pattern of 0:

  The sum of 0 and a number is the number.
  SO, 0 + 6 = 6

SELF-ASSESSMENT PROGRESS REPORT FOR STUDENTS • FACTS WITH 0, 1, 2, AND 3

Complete each fact, and explain your thinking strategy. Place a check mark beside each fact you have mastered.

☐ 2 + 5 =

☐ 0 + 6 =

☐ 3 + 7 =

☐ 8 + 1 =

☐ 6 + 3 =

☐ 2 + 2 =

☐ 0 + 0 =

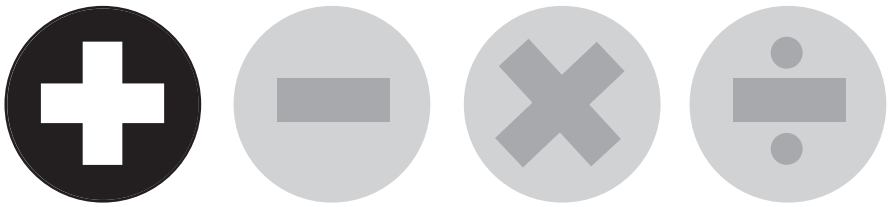
☐ 3 + 3 =

☐ 1 + 9 =

☐ 7 + 2 =

APPENDIX B

PARTNER BINGO



PARTNER BINGO 1

Find a partner.

Answer question 1, and cross out one of the answers on your card (Partner #1).

Have your partner answer question 1, and cross out one of the answers on his/her card (Partner #2).

Take turns answering questions and crossing out one answer each time.

The winner is the first one who has a row, column, or diagonal crossed out.

Partner #1

① 3 + 2 = _____
② 1 + 9 = _____
③ 2 + 8 = _____
④ 1 + 2 = _____
⑤ 6 + 3 = _____
⑥ 5 + 2 = _____
⑦ 4 + 0 = _____
⑧ 2 + 7 = _____
⑨ 4 + 3 = _____
⑩ 3 + 5 = _____
⑪ 1 + 6 = _____
⑫ 7 + 3 = _____
⑬ 8 + 1 = _____
⑭ 4 + 2 = _____
⑮ 2 + 6 = _____
⑯ 0 + 1 = _____

Partner #1

7	8	0	3	9
10	2	9	7	0
6	7	+	10	1
0	2	10	8	4
5	9	4	2	5

Partner #2

① 1 + 4 = _____
② 6 + 2 = _____
③ 3 + 3 = _____
④ 0 + 0 = _____
⑤ 3 + 7 = _____
⑥ 2 + 5 = _____
⑦ 7 + 1 = _____
⑧ 3 + 6 = _____
⑨ 0 + 5 = _____
⑩ 7 + 2 = _____
⑪ 3 + 4 = _____
⑫ 1 + 5 = _____
⑬ 2 + 3 = _____
⑭ 9 + 1 = _____
⑮ 5 + 3 = _____
⑯ 1 + 1 = _____

Partner #2

9	1	10	8	7
2	4	3	5	9
5	8	+	7	3
10	6	0	1	5
3	9	8	4	6

PARTNER BINGO 2

Find a partner.

Answer question 1, and cross out one of the answers on your card (Partner #1).

Have your partner answer question 1, and cross out one of the answers on his/her card (Partner #2).

Take turns answering questions and crossing out one answer each time.

The winner is the first one who has a row, column, or diagonal crossed out.

Partner #1

① 2 + 5 = _____
② 1 + 3 = _____
③ 3 + 7 = _____
④ 8 + 1 = _____
⑤ 0 + 2 = _____
⑥ 3 + 3 = _____
⑦ 2 + 8 = _____
⑧ 5 + 3 = _____
⑨ 0 + 7 = _____
⑩ 2 + 4 = _____
⑪ 3 + 6 = _____
⑫ 1 + 1 = _____
⑬ 5 + 1 = _____
⑭ 4 + 3 = _____
⑮ 2 + 6 = _____
⑯ 2 + 3 = _____

Partner #2

① 6 + 3 = _____
② 5 + 0 = _____
③ 8 + 2 = _____
④ 3 + 4 = _____
⑤ 2 + 7 = _____
⑥ 2 + 1 = _____
⑦ 7 + 3 = _____
⑧ 1 + 4 = _____
⑨ 2 + 6 = _____
⑩ 0 + 10 = _____
⑪ 8 + 1 = _____
⑫ 3 + 5 = _____
⑬ 2 + 3 = _____
⑭ 1 + 7 = _____
⑮ 5 + 2 = _____
⑯ 2 + 2 = _____

Partner #1

1	0	7	6	2
7	2	3	10	7
8	1	+	9	0
10	5	4	0	6
3	6	8	4	9

Partner #2

1	7	6	5	4
10	3	9	10	6
8	2	+	7	3
5	9	8	6	10
9	10	1	5	8

PARTNER BINGO 3

Find a partner.

Answer question 1, and cross out one of the answers on your card (Partner #1).

Have your partner answer question 1, and cross out one of the answers on his/her card (Partner #2).

Take turns answering questions and crossing out one answer each time.

The winner is the first one who has a row, column, or diagonal crossed out.

Partner #1

① 7 + 2 = _____
② 2 + 1 = _____
③ 3 + 3 = _____
④ 1 + 9 = _____
⑤ 6 + 3 = _____
⑥ 2 + 5 = _____
⑦ 3 + 0 = _____
⑧ 7 + 1 = _____
⑨ 4 + 3 = _____
⑩ 2 + 6 = _____
⑪ 3 + 7 = _____
⑫ 1 + 8 = _____
⑬ 4 + 1 = _____
⑭ 3 + 5 = _____
⑮ 8 + 2 = _____
⑯ 0 + 4 = _____

Partner #2

① 3 + 4 = _____
② 0 + 5 = _____
③ 6 + 1 = _____
④ 1 + 1 = _____
⑤ 7 + 3 = _____
⑥ 2 + 2 = _____
⑦ 1 + 4 = _____
⑧ 5 + 3 = _____
⑨ 2 + 7 = _____
⑩ 3 + 1 = _____
⑪ 5 + 2 = _____
⑫ 3 + 6 = _____
⑬ 0 + 0 = _____
⑭ 2 + 4 = _____
⑮ 6 + 2 = _____
⑯ 2 + 3 = _____

Partner #1

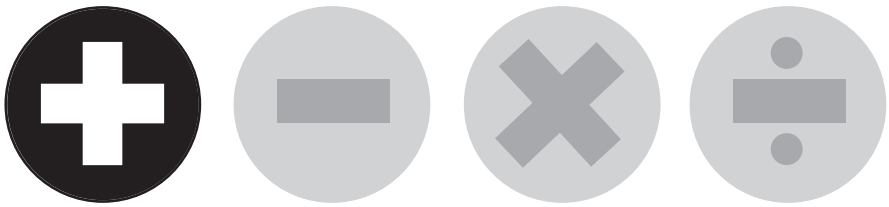
8	1	4	7	10
3	8	5	1	9
9	0	+	10	7
2	6	10	9	6
6	8	3	0	2

Partner #2

4	3	0	7	9
10	8	4	5	1
6	5	+	6	2
9	4	5	1	7
1	7	3	8	5

APPENDIX C

CHALLENGE FACTS



Challenge Facts 1

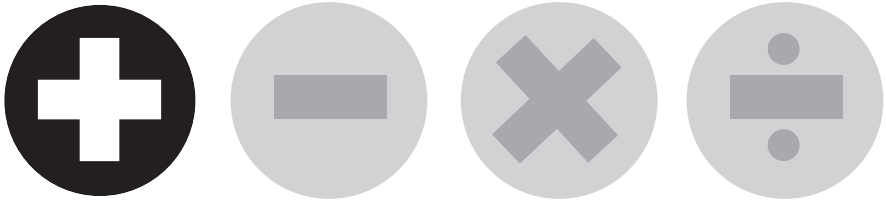
5 + 2	8 + 1	3 + 6	1 + 3	2 + 4	5 + 0	3 + 3	7 + 2
3 + 7	2 + 2	1 + 4	2 + 6	0 + 9	3 + 2	7 + 1	4 + 3
4 + 1	5 + 3	2 + 8	0 + 0	7 + 3	1 + 2	6 + 2	1 + 8
6 + 0	2 + 7	1 + 1	3 + 4	1 + 5	6 + 3	4 + 2	0 + 2
2 + 3	1 + 0	2 + 5	1 + 9	8 + 2	0 + 7	3 + 5	6 + 1

Challenge Facts 2

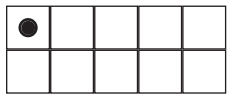
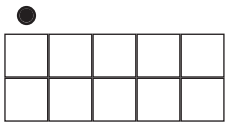
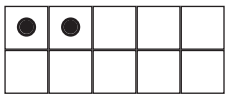
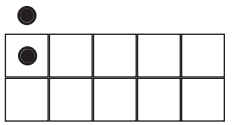
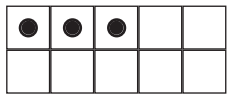
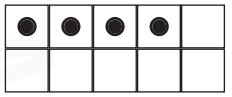
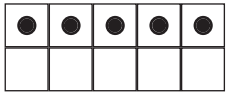
6 + 1	7 + 3	2 + 2	4 + 0	3 + 3	1 + 1	2 + 8	1 + 3
0 + 8	2 + 1	4 + 3	1 + 7	2 + 6	7 + 0	4 + 2	3 + 5
5 + 2	0 + 1	7 + 2	6 + 3	10 + 0	3 + 2	1 + 6	8 + 1
2 + 7	3 + 4	1 + 9	2 + 5	0 + 0	4 + 1	3 + 7	2 + 4
3 + 6	8 + 2	5 + 1	2 + 3	1 + 8	6 + 2	5 + 3	0 + 7

APPENDIX D

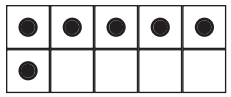
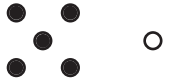
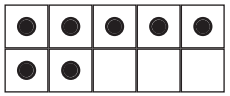
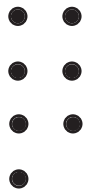
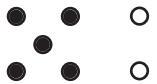
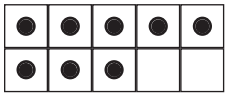
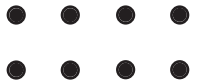
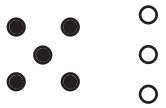
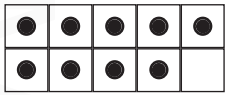
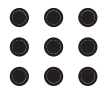
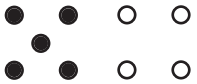
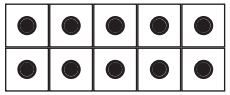
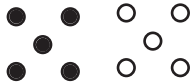
PLAYING CARDS



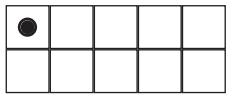
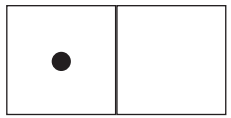
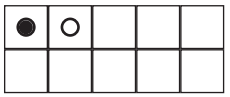
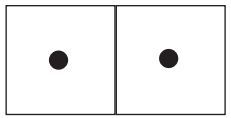
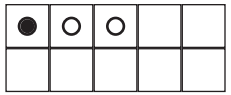
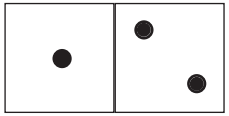
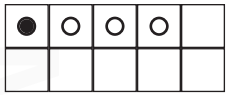
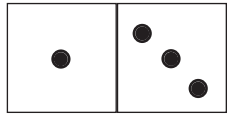
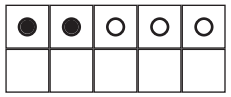
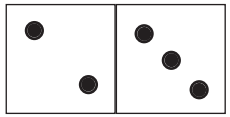
Level 1A

			1			
2				3		
	4				5	

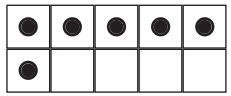
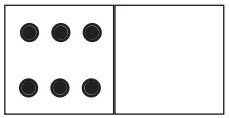
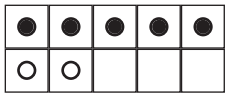
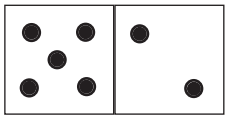
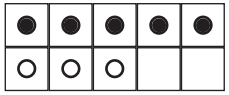
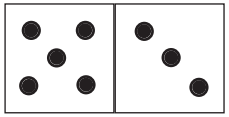
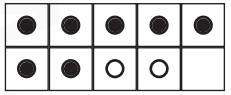
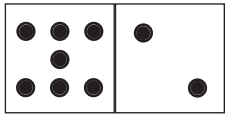
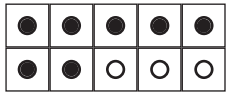
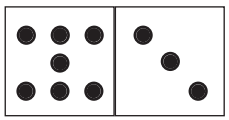
Level 1A

			6			
7				8		
	9				10	

Level 1

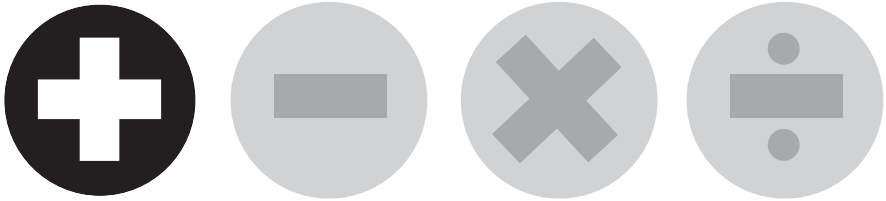
		$\begin{array}{r} 1 \\ +0 \\ \hline \end{array}$	1			$\begin{array}{r} 1 \\ +1 \\ \hline \end{array}$
2			$\begin{array}{r} 1 \\ +2 \\ \hline \end{array}$	3		
$\begin{array}{r} 1 \\ +3 \\ \hline \end{array}$	4			$\begin{array}{r} 2 \\ +3 \\ \hline \end{array}$	5	

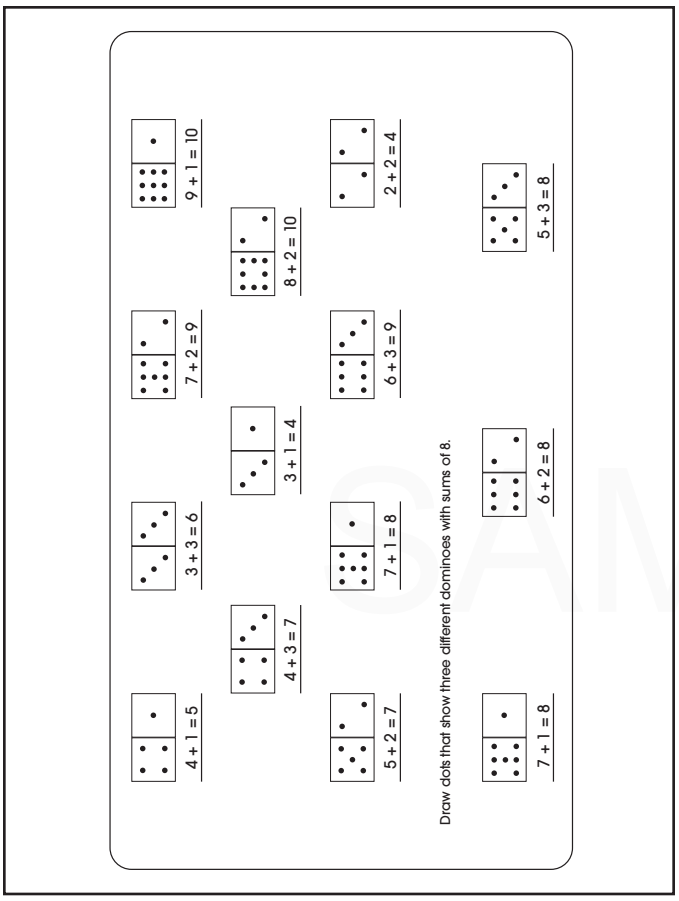
Level 1

		$\begin{array}{r} 6 \\ +0 \\ \hline \end{array}$	6			$\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$
7			$\begin{array}{r} 5 \\ +3 \\ \hline \end{array}$	8		
$\begin{array}{r} 7 \\ +2 \\ \hline \end{array}$	9			$\begin{array}{r} 7 \\ +3 \\ \hline \end{array}$	10	

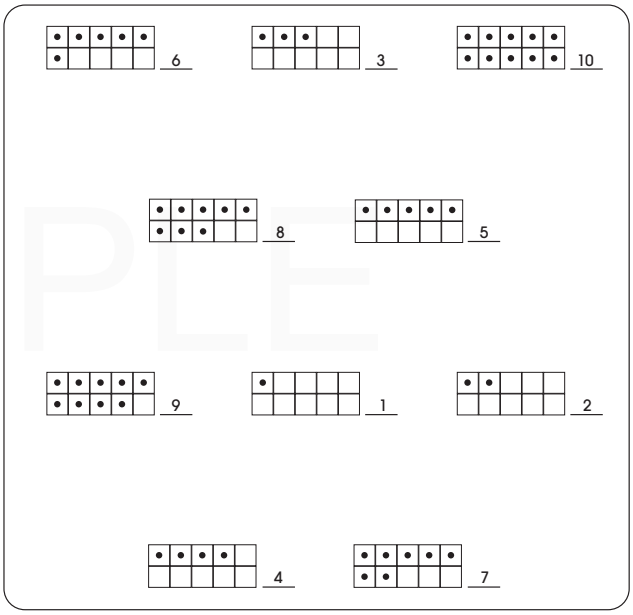
APPENDIX F

ANSWER KEYS

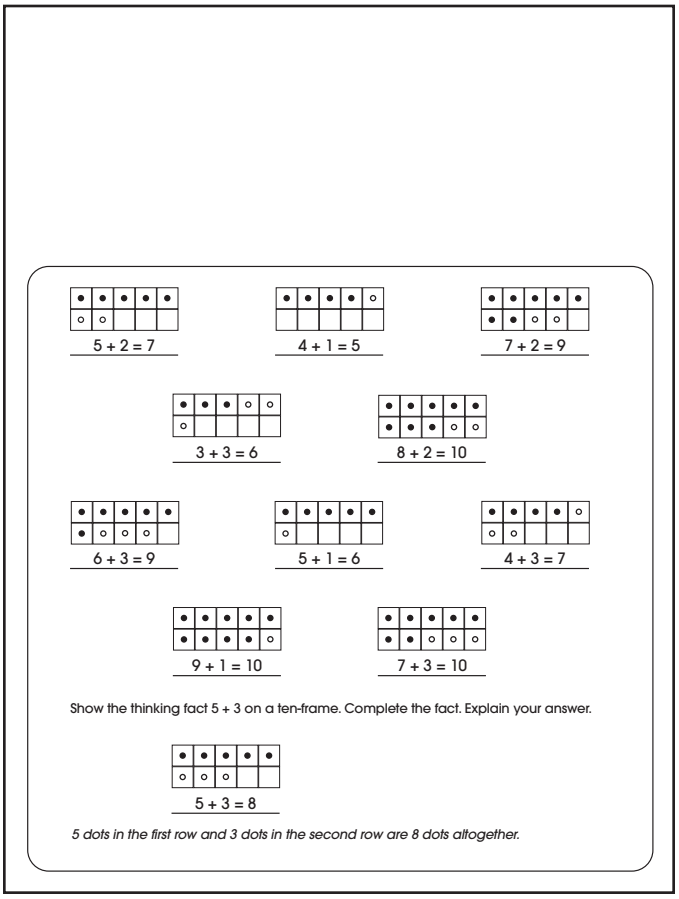




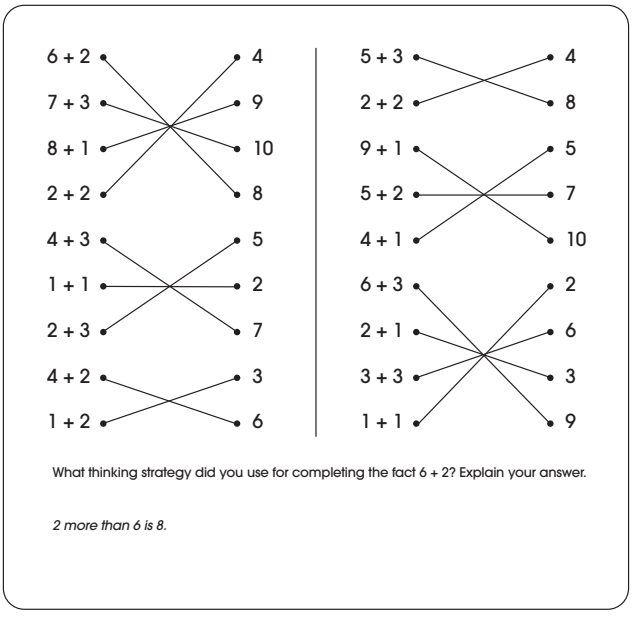
Lesson 1A, page 9



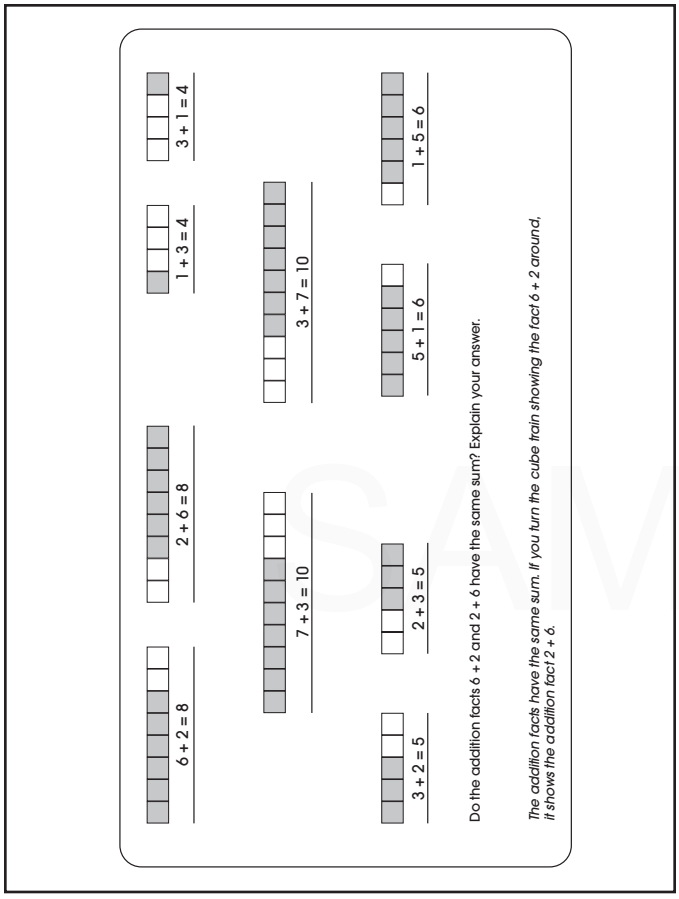
Lesson 1B, page 11



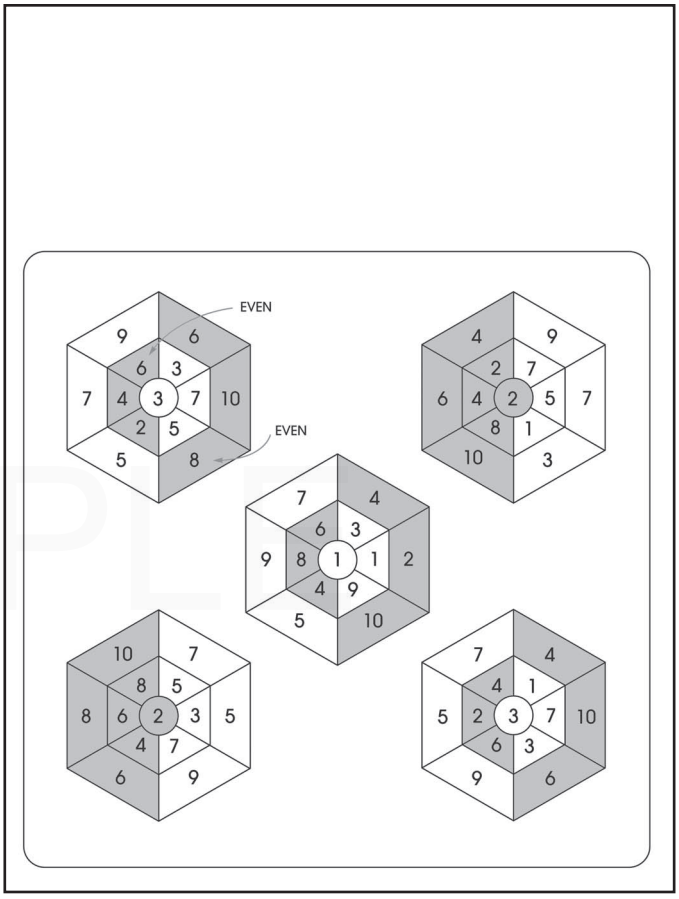
Lesson 1C, page 13



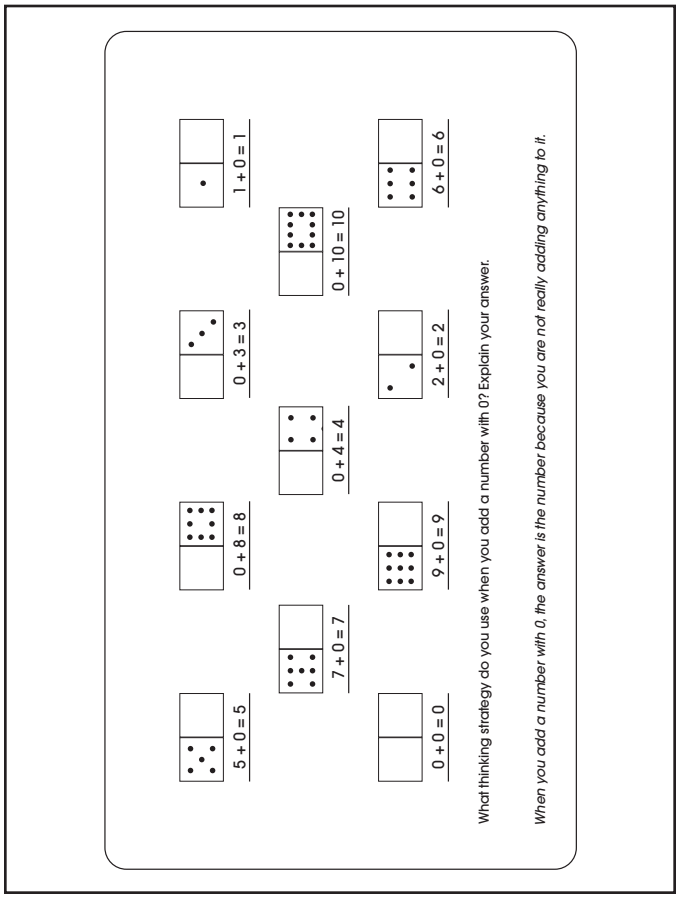
Lesson 1D, page 15



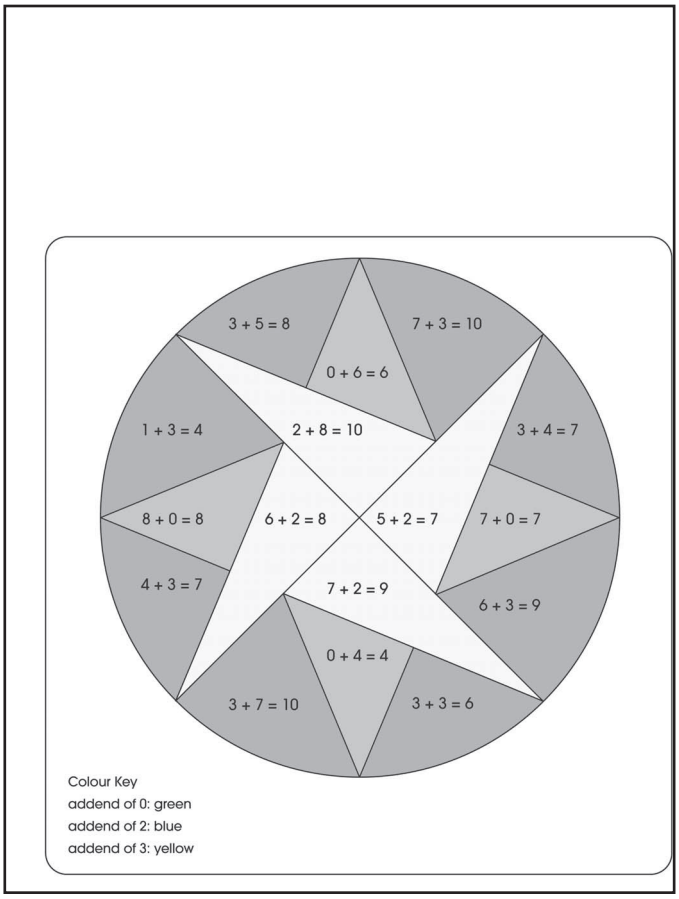
Lesson 1E, page 17



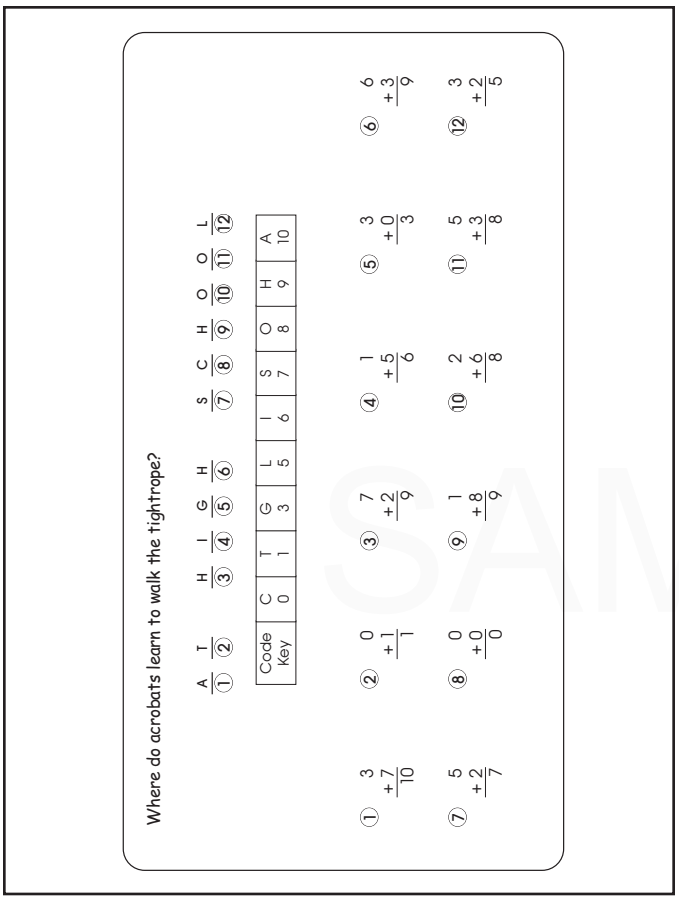
Lesson 1F, page 19



Lesson 1G, page 21



Lesson 1H, page 23



Where do acrobats learn to walk the tightrope?

Code	C	T	H	I	G	L	I	S	C	H	O	O	L
Key	0	1	③	④	⑤	⑥	6	7	⑧	⑨	⑩	⑪	⑫

A	T	H	I	G	L	I	S	C	H	O	O	L
①	②	③	④	⑤	⑥	6	7	⑧	⑨	⑩	⑪	⑫

A
10

① 3 + 7 = 10

② 0 + 1 = 1

③ 7 + 2 = 9

④ 1 + 5 = 6

⑤ 3 + 0 = 3

⑥ 6 + 3 = 9

⑦ 5 + 2 = 7

⑧ 0 + 0 = 0

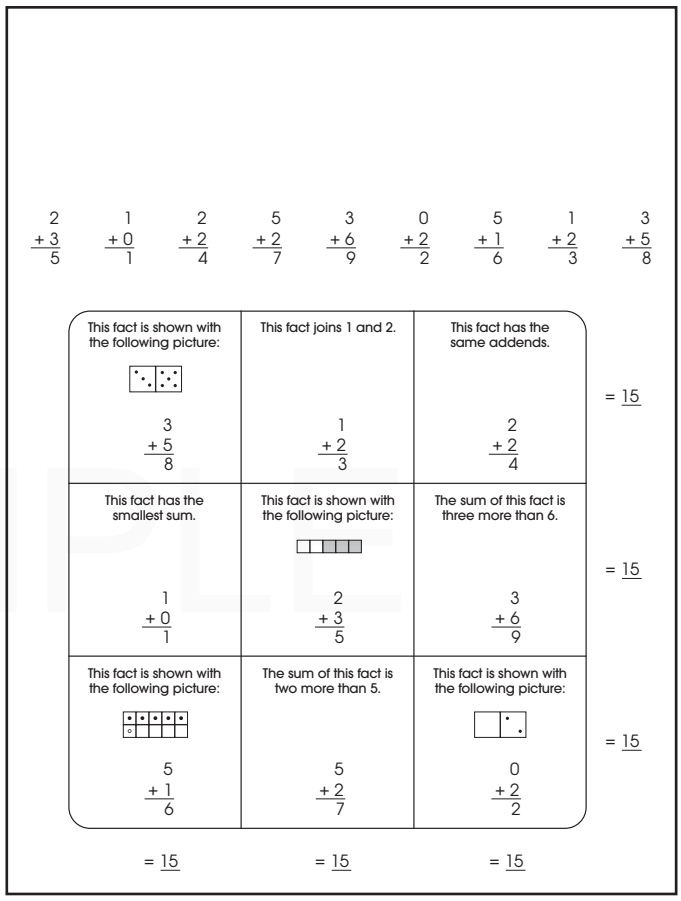
⑨ 1 + 8 = 9

⑩ 2 + 6 = 8

⑪ 5 + 3 = 8

⑫ 3 + 2 = 5

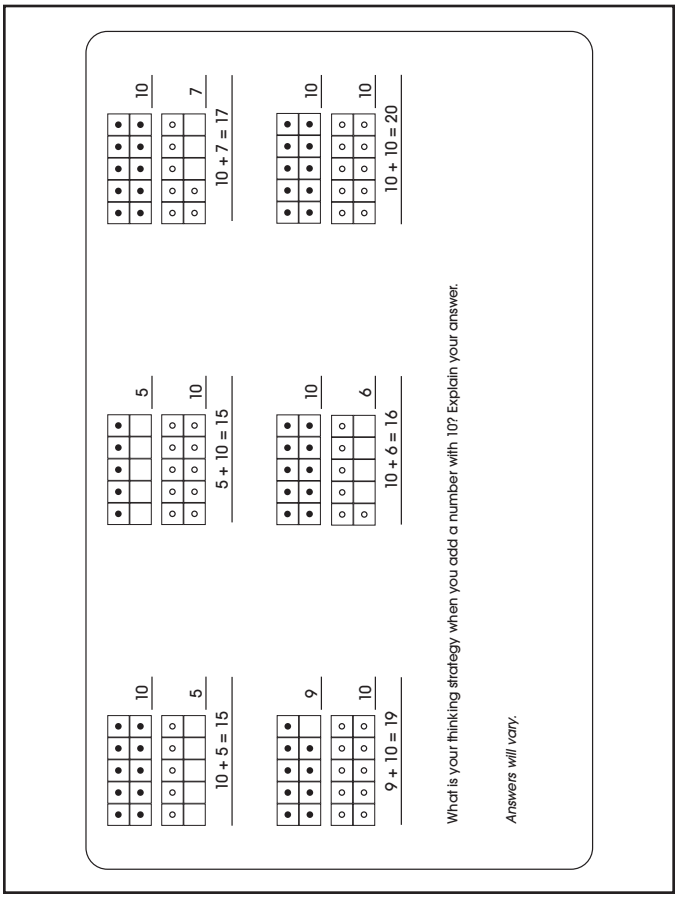
Lesson 1I, page 25



2 + 3 = 5 | 1 + 0 = 1 | 2 + 2 = 4 | 5 + 2 = 7 | 3 + 6 = 9 | 0 + 2 = 2 | 5 + 1 = 6 | 1 + 2 = 3 | 3 + 5 = 8

This fact is shown with the following picture: 3 + 5 = 8	This fact joins 1 and 2. 1 + 2 = 3	This fact has the same addends. 2 + 2 = 4	= 15
This fact has the smallest sum. 1 + 0 = 1	This fact is shown with the following picture: 2 + 3 = 5	The sum of this fact is three more than 6. 3 + 6 = 9	= 15
This fact is shown with the following picture: 5 + 1 = 6	The sum of this fact is two more than 5. 5 + 2 = 7	This fact is shown with the following picture: 0 + 2 = 2	= 15
= 15	= 15	= 15	

Lesson 1J, page 27

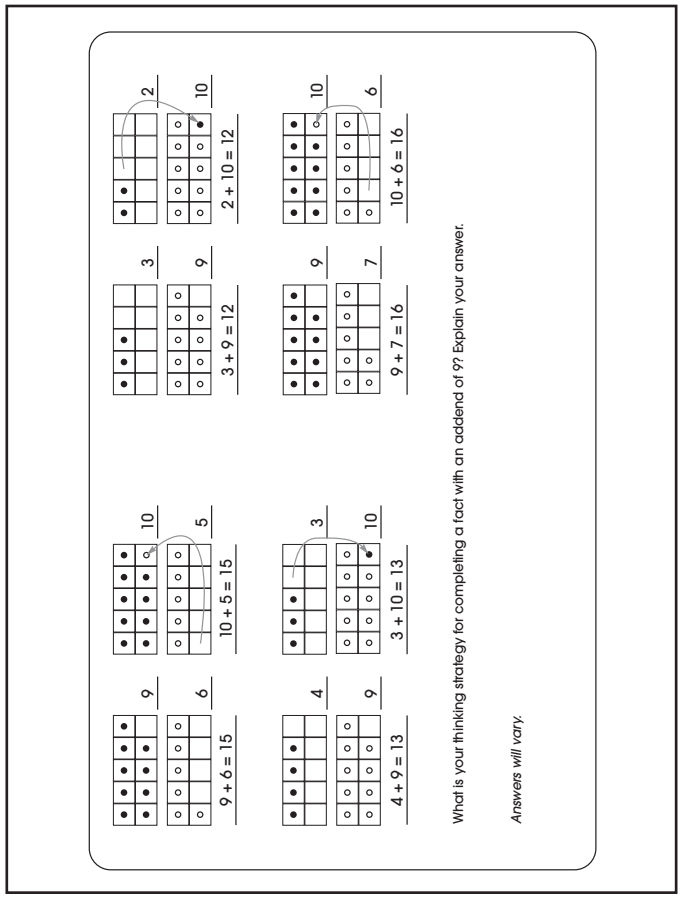


10 + 5 = 15 | 10 + 7 = 17 | 9 + 10 = 19 | 10 + 10 = 20 | 5 + 10 = 15 | 10 + 6 = 16

What is your thinking strategy when you add a number with 10? Explain your answer.

Answers will vary.

Lesson 2A, page 33



2 + 10 = 12 | 3 + 9 = 12 | 10 + 5 = 15 | 3 + 10 = 13 | 9 + 6 = 15 | 4 + 9 = 13 | 10 + 6 = 16 | 9 + 7 = 16

What is your thinking strategy for completing a fact with an addend of 9? Explain your answer.

Answers will vary.

Lesson 2B, page 35